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Handbook of Elementary Physics

Н. И. КОШКИН и М. Г. ШИРКЕВИЧ

СПРАВОЧНИК ПО ЭЛЕМЕНТАРНОЙ ФИЗИКЕ

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N. I. KOSHKIN, M. G. SHIRKEVICH

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PREFACE
TO THE FIFTH RUSSIAN EDITION

All the chapters of the fifth Russian edition (which the present third English edition corresponds to), especially the sections "Fundamental Concepts and Laws", have been considerably revised and supplemented. This made it possible to give a stricter definition of the physical concepts, and also formulate the main laws.

An introduction has been added giving the concept of a vector and information on the systems of units.

The chapters "Mechanics" and "Electricity", unlike the previous editions, consider vector quantities such as the momentum of a body, the angular momentum, and the magnetic moment.

The chapter "Structure of the Atom and Elementary Particles" has been completely revised. The introductory part has been expanded, the number of tables and graphs increased, and the data have been up-dated.

The authors express their gratitude to M. I. Bludov for a number of valuable remarks which have been taken into account in preparing the present edition.

The handbook has been revised by N. I. Koshkin.

N. I. Koshkin and M. G. Shirkevich

PREFACE
TO THE FIRST RUSSIAN EDITION

The present handbook covers all the main branches of elementary physics. Each chapter (or section of a chapter) consists of two parts. The first gives a brief treatment of the fundamental concepts and laws, and the second contains reference tables and graphs.

The theoretical information set out in the first part by no means claims completeness. Here we have given the main definitions and formulations of laws, which are sometimes accompanied by brief explanations and examples. The information given cannot even in a remote form replace a textbook or a special guide in physics.

The tables and graphs included in the handbook also do not claim completeness of covering all the reference information on the relevant branch of physics. An attempt has been made to condense the multitude of information available at present into data most frequently used today by specialists in industry and agriculture.

Special attention has been given to the choice of data on the latest developments in physics such as semiconductors, ferroelectrics and nuclear physics.

The authors express their profound gratitude to V. F. Yakovlev, V. I. Rodichev, and M. A. Gorbunov, of the N. K. Krupskaya Moscow District Pedagogical Institute, for their helpful advice and the aid they rendered in compiling this book.

N. I. Koshkin and M. G. Shirkevic

EXPLANATORY NOTES

Most of the tables are arranged in alphabetical order. Some, however, are arranged in the order of increasing or decreasing values of the tabulated quantity.

The numerical values of the quantities are given to two or three significant digits after the decimal point, which is sufficiently precise for most technical calculations.

The number of digits given after the decimal point varies in the tables. This is due to the circumstance that some substances can be obtained in the pure form, whereas others are complicated mixtures of substances. For example, the density of platinum is given to four significant digits: 21.46, whereas that of brass is given to within three units of the second significant digit: 8.4-8.7, since the density of brass varies within these limits depending on the composition of the given specimen.

If the heading of a column in a table contains a factor such as 10^n, this denotes that the values of the quantity in that column have been decreased 10^n times; hence, to find the true values one must *multiply* the values given in the table by this factor. For example, in the heading of the last column in Table 18: "Compressibility of Selected Liquids at Different Temperatures" (p. 58) the compressibility β has the factor 10^{-6} ($\beta \times 10^{-6}$ atm^{-1}). Thus, the compressibility of acetone (at 14.2 °C), according to this table, is 111×10^{-6} atm^{-1}.

The notes to the tables give the conditions for which the values of the tabulated quantities are valid (if these conditions are not indicated completely in the heading of the table), as well as additional information on how to use the tables, etc.

If the physical significance of the tabulated quantity is not quite clear to the reader, he should refer to the relevant section: "Fundamental Concepts and Laws". The appendices contain information on the units of physical quantities, formulas for approximate calculations, and the values of selected universal physical constants.

INTRODUCTION

Vectors and Scalars

All quantities in physics are divided into two kinds: scalar and vector. Quantities whose determination requires that we know only their numerical values are called *scalar quantities* or *scalars*. Quantities whose determination requires that we know not only their numerical values, but also their direction, are called *vector quantities* or *vectors*.

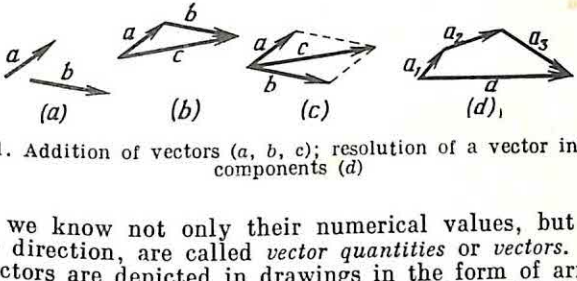

Fig. 1. Addition of vectors (*a*, *b*, *c*); resolution of a vector into its components (*d*)

Vectors are depicted in drawings in the form of arrows; the length of an arrow equals (to a definite scale) the numerical value (module) of the vector, and its direction corresponds to that of the vector. Two vectors are equal if they have equal modules and the same directions. The addition of vectors is shown in Fig. 1. Two vectors *a* and *b* (Fig. 1*a*) can be added, for example, in two ways.

In the first way (Fig. 1*b*), vectors *a* and *b* are transferred parallel to themselves so that the end of one vector coincides with the beginning of the other. Now a vector drawn from the beginning of the first vector to the end of the second (*c* in Fig. 1*b*) equals the sum of these vectors. It is called a *vector sum*.

In the second way (Fig. 1*c*), vectors *a* and *b* are transferred parallel to themselves so that their beginnings coincide.

The sum of the vectors will equal the diagonal of a parallelogram constructed on them (in Fig. 1*c* the sum of the vectors equals *c*). Vectors are therefore sometimes said to be added according to the parallelogram law. Every vector *a* (Fig. 1*d*) can be resolved into several vectors a_1, a_2, etc., whose sum gives vector *a*. The replacement of one vector with the sum of other vectors is called *resolution of the vector into its components*. For example, the components of vector *a* in Fig. 1*d* are vectors a_1, a_2 and a_3. Multiplication of a vector by a positive scalar yields a vector having the same direction, but a different module. Multiplication of a vector by a negative scalar reverses its direction.

Systems of Units

To measure a physical quantity means to compare it with another taken as a unit. The quantity being measured and the unit used for measurement should be *homogeneous*, i.e. determine the same property of bodies, but differ only in their numerical value.

A *unit* is a value of a physical quantity used as the basis in comparing homogeneous quantities. Basic and derived units are distinguished. The value of a *basic unit* is chosen independently of the values of other quantities; the value of a *derived unit* is determined by the physical relationship between the quantity it measures and other quantities. A complex of basic and derived units between which definite relationships exist is called a *system of units*.

Two systems are mainly used in physics—the International System of Units (SI) and the CGS system. The USSR State Standard GOST 9867-61 recommends the SI system as preferable.

The basic units of the SI system are the metre (m)—the unit of length, the kilogram (kg)—the unit of mass, the second (s)—the unit of time, the ampere (A)—the unit of current intensity, the kelvin (K)—the unit of temperature, and the candela (cd)—the unit of luminous intensity.

The *metre* is the length equal to 1 650 763.73 wavelengths in vacuum of the radiation of the krypton-86 atom corresponding to orange light (i.e. to the transition between the levels 5*d* and 2*p*, see p. 244).

The *kilogram* is the mass of the international prototype of the kilogram.

The *second* is $\frac{1}{31\,556\,925.974\,7}$ part of a conditional tropical year (for uniformly running time). According to a decision of the 13th General Conference on Weights and Measures (1967) and the relevant USSR State Standard, the second is the duration of 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.

The definitions of the other basic quantities are given in the text: the *ampere* on p. 165, the *kelvin* on p. 66, and the *candela* on p. 196.

The basic units in the CGS system are the *centimetre* (cm) (10^{-2} m), the *gramme* (g) (10^{-3} kg), and the *second*. The dielectric constant and the permeability of a vacuum are taken equal to unity (see p. 164 and Table VII of the Appendix).

CHAPTER I

MECHANICS

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When a body changes its position relative to other bodies it is said to be in *mechanical motion*. This change in position is determined by a change in the distance between the fixed points of the bodies.

A. KINEMATICS

FUNDAMENTAL CONCEPTS AND LAWS

Kinematics is the study of the motion of bodies without regard to the cause of that motion.

The simplest moving body is a *point mass*, defined as a body whose dimensions can be ignored in describing its motion. For example, the annual motion of the Earth about the Sun can be regarded as the motion of a point mass, whereas the daily revolution of the Earth about its axis cannot.

Every solid body can be regarded as a system of rigidly bound point masses.

The path described by a moving point is called a *trajectory*. According to the form of the trajectory we distinguish between *rectilinear motion* (the trajectory is a straight line) and *curvilinear motion* (the trajectory is a curve). We also distinguish between uniform and non-uniform motion.

1. Rectilinear Motion

Uniform motion is motion in which a body traverses equal distances in equal time intervals; otherwise motion is *non-uniform*. Uniform motion is characterized by its velocity. The *velocity* of uniform motion (v) is the distance (s) traversed in unit time (t):

$$v = \frac{s}{t}$$

or

$$s = vt \tag{1.1}$$

In non-uniform motion we differentiate between instantaneous and average velocities. If a body passes over the distance Δs in the time interval from t_0 to $t_0 + \Delta t$, then

$$v_{\text{av}} = \frac{\Delta s}{\Delta t}$$

is the *average velocity* for the time interval Δt. The *instantaneous velocity*, or the velocity at a given moment t_0, is the limit of the ratio:

$$v_{\text{inst}} = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

Velocity is a vector quantity. It is characterized by magnitude and direction in space. The addition (composition) of velocities is performed according to the parallelogram law (addition of vectors).

The units of velocity are: cm/s (CGS), m/s (SI), and non-system units km/s, and km/h.

Motion in which the velocity receives equal increments in equal time intervals is called *uniformly accelerated*. The rate of change of the velocity in unit time is called the *acceleration* (a); more exactly

$$a = \lim_{t \to t_0} \frac{v_t - v_0}{t - t_0} \tag{1.2}$$

where v_t is the velocity at the time t, and v_0—the velocity at the initial moment of time t_0.

Acceleration is also a vector quantity,

The units of acceleration are: cm/s^2 (CGS), m/s^2 (SI), and non-system unit km/s^2.

The velocity of uniformly accelerated motion at any given moment is determined by the formula:

$$v_t = v_0 + at \tag{1.3}$$

where v_0 is the initial velocity.

The acceleration may be positive (*accelerated motion*) or negative (*decelerated motion*).

The distance traversed during the time t in uniformly accelerated motion is given by the formula:

$$s = v_0 t + \frac{at^2}{2} \tag{1.4}$$

The terminal velocity in uniformly accelerated motion is determined by the initial velocity, the acceleration, and the distance traversed:

$$v_t^2 = v_0^2 + 2as \tag{1.5}$$

The motion of freely falling bodies from a small altitude (much less than the Earth's radius) is an example of rectilinear motion with constant acceleration. If we denote the height from which the body falls ($v_0 = 0$) by h, and the acceleration of free fall (acceleration due to gravity) by g, then

$$h = \frac{gt^2}{2} \tag{1.4a}$$

where t is the duration of falling.

2. Rotational Motion

The *circular motion of a point* about an axis is motion in which the trajectory of the point is a circle whose centre is on the axis and whose plane is perpendicular to the axis. The *rotational motion of a body* about an axis is motion in which all the points of the body describe circular motion about this axis.

Uniform rotation is motion in which a body turns through equal angles in equal time intervals.

The *angular velocity* (ω) of uniform rotation is the angle of rotation in unit time:

$$\omega = \frac{\varphi}{t} \tag{1.6}$$

where φ, measured in radians, is the angle through which the body turns in the time t. The angular velocity may also be expressed in terms of the number of revolutions n in unit time, or the period of revolution T:

$$\omega = 2\pi n \tag{1.7}$$

$$\omega = \frac{2\pi}{T} \tag{1.8}$$

The instantaneous velocity of a point is the *linear velocity* of the point in rotational motion. The velocity vector is tangent to the trajectory. The angular velocity ω is related to the linear velocity v by the formula:

$$v = \omega R \tag{1.9}$$

where R is the distance from the point to the axis of rotation.

In the case of non-uniform rotation we distinguish between instantaneous and average angular velocities. If a body has turned through the angle $\Delta\varphi$ in the time from t_0 to $t_0 + \Delta t$, then the *average angular velocity* (ω_{av}) for the time Δt is

$$\omega_{\text{av}} = \frac{\Delta\varphi}{\Delta t}$$

The limit of this ratio is, by definition, the *instantaneous angular velocity* at the time t_0:

$$\omega_{\text{inst}} = \lim_{\Delta t \to 0} \frac{\Delta\varphi}{\Delta t} \tag{1.10}$$

Rotational motion in which the angular velocity receives equal increments in equal time intervals is called *uniformly accelerated*.

The *angular acceleration* of uniformly accelerated rotation (ε) is the rate of change of the angular velocity in unit time:

$$\varepsilon = \lim_{t \to t_0} \frac{\omega_t - \omega_0}{t - t_0} \tag{1.11}$$

where ω_t is the angular velocity at the time t, and ω_0—the angular velocity at the initial moment of time t_0.

The angular velocity ω and the angular acceleration ε are vector quantities. The direction of the vector ω is determined by the right-hand screw law (Fig. 2): if the head of a screw is rotated in the direction of rotation of a body, then the translational motion of the screw coincides with the direction of ω; the vector ω is directed along the axis of rotatio ω. The direction of the vector of angula ω acceleration coincides with that of ω if the angular velocity grows; when the latter diminishes, the direction of the vector ε is opposite to that of ω.

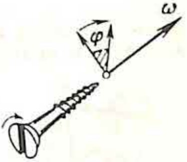

Fig. 2. Determining the direction of an angular velocity vector by the right-hand screw rule

If uniformly accelerated rotational motion is characterized by the number of revolutions n in unit time, then the acceleration ε^* can be introduced:

$$\varepsilon^* = \lim_{t \to t_0} \frac{n_t - n_0}{t - t_0} \tag{1.12}$$

where n_t is the number of revolutions at the time t, and n_0—the number of revolutions at the initial moment of time t_0.

The angular velocity of uniformly accelerated rotation and the number of revolutions in unit time after the time t has elapsed from the beginning of rotation are

$$\omega = \omega_0 + \varepsilon t \text{ and } n = n_0 + \varepsilon^* t \tag{1.13}$$

The angle of rotation in uniformly accelerated rotation and the number of revolutions are

$$\varphi = \omega_0 t + \frac{\varepsilon t^2}{2} \quad \text{and} \quad N = n_0 t + \frac{\varepsilon^* t^2}{2} \tag{1.14}$$

The velocity of a body travelling along a curvilinear trajectory varies not only in magnitude, but also in direction (Fig. 3).

The change in magnitude of the linear velocity in unit time is called the *tangential acceleration*:

$$a_{\tan}=\frac{v_t-v_0}{t-t_0}=\frac{\Delta v_{\tan}}{\Delta t} \qquad (1.15)$$

or, more strictly,

$$a_{\tan}=\lim_{\Delta t\to 0}\frac{\Delta v_{\tan}}{\Delta t} \qquad (1.16)$$

where v_t and v_0 are the linear velocities at the time $t_0+\Delta t$ and at the initial moment of time t_0. The tangential acceleration is related to the angular acceleration by the formula:

$$a_{\tan}=\varepsilon R \qquad (1.17)$$

The direction of $a_{\tan}$ at a given point of a trajectory coincides with that of the linear velocity or is opposite to it.

The change in velocity in direction in unit time is called the *normal acceleration*:

$$a_{\rm n}=\frac{\Delta v_{\rm n}}{\Delta t} \qquad (1.18)$$

or, more strictly,

$$a_{\rm n}=\lim_{\Delta t\to 0}\frac{\Delta v_{\rm n}}{\Delta t} \qquad (1.19)$$

The normal acceleration is directed along the radius of curvature of a trajectory (towards the axis of rotation).

For example, when a body is in uniform rotation, its points are in accelerated motion because the direction of their velocity is continually changing. The normal acceleration in this case is directed towards the axis of rotation

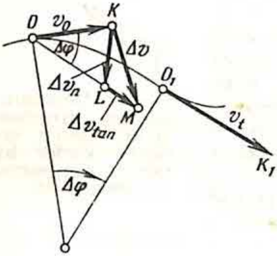

Fig. 3. Tangential and normal acceleration.
Vector $OK=v_0$ determines the velocity at point O (at the time t_0); vector $O_1K_1=v_t$ determines the velocity at point O_1 (at the time $t_0+\Delta t$). Vector v_t is transferred parallel to itself to point O (line segment OM). The total change in the velocity Δv is determined by line segment KM; the change in the velocity with respect to its module (the tangential change in the velocity $\Delta v_{\rm tan}$) equals line segment LM ($OK=OL$); the change in the normal component of the velocity $\Delta v_{\rm n}$ equals line segment KL. The accelerations a, $a_{\rm tan}$, and $a_{\rm n}$ will equal the corresponding changes in the velocity in unit time

(i.e. perpendicular to the direction of the linear velocity) and is called the *centripetal acceleration*:

$$a_c = \frac{v^2}{R} = \omega^2 R \qquad (1.20)$$

where v is the linear velocity, ω—the angular velocity, and R—the radial distance of the point from the axis of rotation.

The total acceleration of a point of a body in uniformly accelerated rotational motion (see Fig. 3) is

$$a^2 = a_n^2 + a_{tan}^2 \qquad (1.21)$$

3. Motion of Bodies in the Earth's Gravitational Field

Figure 4 illustrates the trajectories of bodies which are projected from point A near the surface of the Earth with different velocities.* In all cases the velocity is directed horizontally. The trajectory is a circle if the velocity v of a body at point A is such that the acceleration of free fall g is equal to the centripetal acceleration $\frac{v^2}{R}$ (R is the radius of the trajectory, which for a small altitude can be taken equal to the radius of the Earth). Hence

$$v = \sqrt{Rg} \approx 7.93 \text{ km/s}$$

This value is called the *first cosmic velocity*. If the velocity of the body at point A is greater than 7.93 km/s but less than 11.16 km/s, then the trajectory is an ellipse; the focus of the ellipse nearer to the point of departure (point A) lies at the centre of the Earth (this ellipse is depicted by a solid line in Fig. 4). If the velocity of the body equals

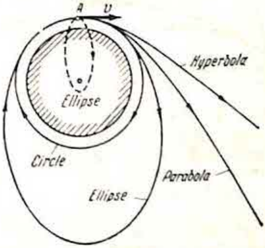

Fig. 4. Trajectories of bodies in the Earth's gravitational field

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* The resistance of the air is ignored.

11.16 km/s—the *second cosmic velocity*, then the trajectory is a parabola. If the initial velocity is greater than 11.16 km/s, then the trajectory becomes a hyperbola. In the last two cases the body leaves the Earth and goes off into interplanetary space. The least velocity required for a body to leave the Earth is sometimes called the *escape velocity*. The path of a body moving with a velocity less than 7.93 km/s is an arc of an ellipse (dotted line in Fig. 4) whose distant focus coincides with the centre of the Earth.

If the velocity is much less than 7.93 km/s, the arc may be regarded as parabolic.

If a body is projected from the surface of the Earth at the angle α, with an initial velocity v_0 much less than $v = 7.93$ km/s, then in this case, too, the acceleration of free fall may be considered constant in magnitude and direction, while the surface of the Earth may be regarded as flat. The trajectory will then be a parabola (Fig. 5). The range (S) and the maximum height (H) are calculated by the formulas:

$$S = \frac{v_0^2 \sin 2\alpha}{g} \quad \text{and}$$

$$H = \frac{v_0^2 \sin^2 \alpha}{2g} \qquad (1.22)$$

where v_0 is the initial velocity of the body.

The range will be the same for two values of the angle of projection: α_1 and α_2, where $\alpha_2 = 90° - \alpha_1$.

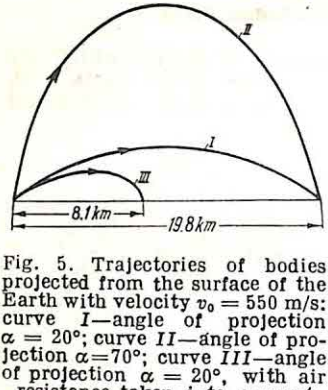

Fig. 5. Trajectories of bodies projected from the surface of the Earth with velocity $v_0 = 550$ m/s: curve *I*—angle of projection $\alpha = 20°$; curve *II*—angle of projection $\alpha = 70°$; curve *III*—angle of projection $\alpha = 20°$, with air resistance taken into account

The maximum range corresponds to the angle $\alpha = 45°$.

If the resistance of the air is taken into account, the range and the height of the trajectory diminish. For example, in the absence of air resistance a body thrown at an angle $\alpha = 20°$ with an initial velocity $v_0 = 550$ m/s would have a range of 19.8 km, whereas a projectile fired at the same angle and with the same initial velocity in air would have a range of only 8.1 km.

TABLES

Table 1

Accelerations (Approximate Values)

Accelerated motion	Acceleration, m/s^2	Decelerated motion	Deceleration, m/s^2
Subway train . . .	1	Emergency braking of automobile . .	4-6
Racing car	4.5	Landing jet plane	5-8
Elevator in house	0.9-1.6	Parachute opening when rate of fall is 60 m/s	about 60
Passenger train . .	0.35		
Tramcar	0.6		
Launching of rocket	30-90		
Shell in gun barrel	100 000		

Table 2

Kinematic Parameters of the Planets

Planet	T_S (in years)	T_0	v_0, km/s	v, km/s	N
Mercury . .	0.241	88 days	48.8	4.20	—
Venus . . .	0.615	247±5 days	35.0	10.2	—
Earth . . .	1.00004	23 h 56 min 4 s	29.8	11.16	1
Mars	1.881	24 h 37 min 23 s	24.2	5.01	2
Jupiter . .	11.86	9 h 51 min	13.06	59.5	12
Saturn . . .	29.46	10 h 14 min	9.65	35.4	9
Uranus . .	84.01	10 h 49 min	6.78	22.2	5
Neptune . .	164.8	14 h (?)	5.42	24.8	2
Pluto . . .	247.7	—	4.73	—	—
Moon . . .	(satellite of the Earth)	27 days 7 h 43 min 11 s	—	2.37	—

Note. T_S—period of revolution about the Sun
T_0—period of revolution about axis
v_0—orbital velocity
v—escape velocity
N—number of satellites.

Table 3

Dependence of Escape Velocity v on the Altitude H above the Earth's Surface

H, 10^3 km	v, km/s	H, 10^3 km	v, km/s	H, 10^3 km	v, km/s
0	11.19	5	8.37	40	4.15
0.5	10.77	10	6.98	50	3.76
1	10.40	20	5.50		
2	9.76	30	4.68		

Table 4

Period of Revolution T of Earth's Satellite at Different Altitudes H

H, km	T, h	H, km	T, h	H, km	T, h
0	1.41	1000	1.75	5 000	3.35
250	1.49	1500	1.93	10 000	5.78
500	1.58	1690	2.00	35 800*	23.935
750	1.68	2000	2.12		

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* The mean altitudes of revolution are indicated.

B. DYNAMICS

FUNDAMENTAL CONCEPTS AND LAWS

Dynamics deals with the laws of motion of bodies and the factors which cause or change this motion. Any change in the motion or shape of a body is a result of the interaction of at least two bodies.

The physical quantity characterizing the interaction of bodies is called a *force*; it determines the change of motion, or the change of shape of a body, or both.

Force is a vector quantity. The addition of two forces simultaneously acting on a body is performed according to the parallelogram law (addition of vectors).

1. Laws of Dynamics

Newton's first law. Every body continues to be in a state of rest or straight-line uniform motion unless it is compelled to change that state by the application of some external force.

That property of matter by virtue of which a body tends to retain the magnitude and direction of its velocity unchanged when no (or balanced) forces act on it is called *inertia*. The change in the motion of a body depends not only on the force acting on it, but also on the properties of the body itself. Experiments show that the ratio of the magnitude of a force to the acceleration it produces always remains constant for a given body (at low velocities). The physical quantity proportional to this ratio is called the *mass* of a given body:

$$m = \frac{kF}{a}$$

The coefficient k depends on the chosen system of units. Mass is a measure of the inertia of a body.

The mass in Newton's laws characterizes inert properties; mass is also encountered in the law of universal gravitation (see p. 35), where it characterizes gravitational properties, i.e. the property of bodies to attract each other. Consequently, we can distinguish between the *inert mass* and the *gravitational mass*. All experimental facts, however, indicate that these two masses of a body are equal. Therefore simply a mass is spoken of in physics.

At high velocities close to the velocity of light in a vacuum, the mass depends on the velocity:

$$m = \frac{m_0}{\sqrt{1-\beta^2}} \qquad (1.23)$$

where m is the mass of a moving body, m_0—that of a stationary body, $\beta = \frac{v}{c}$, v—the velocity of the moving body, and c—the velocity of light in a vacuum.

Newton's second law. The acceleration of a body as a result of the force F acting on it is proportional to the magnitude of this force and inversely proportional to the mass of the body m. The direction of the acceleration coincides with that of the force:

$$a = k\frac{F}{m} \tag{1.24}$$

The unit of force or mass is selected so that the coefficient k equals unity.

In the SI system the unit of force is the force which imparts an acceleration of 1 m/s^2 to a body having a mass of 1 kg. This unit is called the *newton* (N). In the CGS system the unit of force is the *dyne* (dyn);

$$1\ \mathrm{N} = 10^5\ \mathrm{dyn}$$

The product of the mass of a body and its velocity is called the *impulse* of the body (or its *momentum*):

$$p = mv$$

Impulse is a vector quantity coinciding in direction with velocity.

If the force applied to a body is constant in magnitude and direction, Newton's law can be written as follows, using Eq. (1.2):

$$F = \frac{mv_t - mv_0}{t - t_0} \quad \text{or} \quad F = \frac{\Delta p}{\Delta t} \tag{1.25}$$

Equation (1.25) can be written more strictly as follows:

$$F = \lim_{\Delta t \to 0} \frac{\Delta p}{\Delta t} \tag{1.26}$$

Thus, the change in the impulse of a body in unit time equals the acting force (both in magnitude and direction).

Newton's third law. The forces with which two bodies act on each other are directed along one straight line, are equal in magnitude and opposite in direction:

$$F_1 = -F_2$$

or

$$m_1a_1 = -m_2a_2 \tag{1.27}$$

where F_1 is the force acting on the first body, F_2—the force acting on the second body, and m_1 and m_2—the masses of the first and second bodies, respectively.

The forces acting on a system of bodies can be divided into two kinds—internal and external. The forces acting between the bodies in the system are called *internal*, and the forces due to interaction with bodies not belonging to the given system are called *external*. A system is called *closed* if external forces are absent. The *law of conservation of impulse* (*momentum*) is observed for a closed system: the vector sum of the impulses of bodies in a closed system is a constant quantity:

$$\sum p_i = \text{const}$$

For example, in a system consisting of two bodies the following relation is satisfied:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

where v_1 and v_2 are the velocities of the first and second bodies before interaction, and u_1 and u_2—the respective velocities after interaction.

2. Dynamics of Rotation

Newton's second law for rotational motion takes the form:

$$M = J\varepsilon \qquad (1.28)$$

Here the moment of inertia (J) is analogous to the mass, the moment of force (M)—to the force, and the angular acceleration (ε)—to the linear acceleration.

The *moment of force* is the product of the force and its arm (the shortest distance between the line of action of the force and the axis of rotation).

The moment of force is a vector quantity; its direction is determined according to the right-hand screw rule (see below). The directions of the moment of a force and the angular acceleration coincide.

If two moments of force are applied to a body, producing rotation in opposite directions, then one of them is arbitrarily considered positive, and the other negative.

The *moment of inertia* (or *rotary inertia*) of a point mass about an axis is equal to the product of the mass and the square of its distance from the axis:

$$J = mR^2 \qquad (1.29)$$

The moment of inertia of a body is the sum of the moments of inertia of the point masses which the body is composed of. The moment of inertia of a body can be expressed in terms of its mass and dimensions.

The moment of inertia of a body about an axis can be determined if we know the moment of inertia of the body about a parallel axis passing through the centre of gravity of the body (see p. 46), the mass of the body m, and the distance between the two axes b:

$$J = J_0 + mb^2 \tag{1.30}$$

In uniform rotational motion the sum of all the moments of the forces acting on the body is equal to zero.

Newton's second law for rotational motion can be expressed through the impulse of a body. To do this a quantity called the moment of momentum or the angular momentum L is used. The *angular momentum* is a vector quantity numerically equal to the product of the impulse of a body (momentum) and the arm r:

$$L = mvr = pr$$

The direction of the vector L is determined according to the right-hand screw rule: if the head of a screw is turned in the direction of rotation of a body, then the forward motion of the screw will coincide with the direction of L. The directions of M and ω are determined similarly. Hence the law of rotational motion can be defined as follows: the change in angular momentum in unit time equals the moment of the forces acting on a body:

$$M = \frac{L_t - L_0}{t - t_0} \quad \text{or} \quad M = \frac{\Delta L}{\Delta t} \tag{1.31}$$

where L_t and L_0 are the angular momenta at the time t and t_0.

The vector sum of the angular momenta of bodies forming a closed system is a constant quantity:

$$\sum L_i = \text{const}$$

(the law of conservation of angular momentum).

The uniform motion of a point in a circular path (uniform circular motion) is characterized by centripetal acceleration (which causes the velocity to change in direction) and can take place only if a force acts to produce this acceleration. This force is applied to the point which is describing circular motion and is called the *centripetal force*:

$$F_c = \frac{mv^2}{R} = m\omega^2 R \tag{1.32}$$

The centripetal force is directed along the radius towards the axis of rotation and its moment is equal to zero (the arm of the force is zero).

The units of the moment of force are N·m (SI) and dyn·cm (CGS); the units of the moment of inertia—kg·m² (SI) and g·cm² (CGS);

$$1\ \text{kg}\cdot\text{m}^2 = 10^7\ \text{g}\cdot\text{cm}^2$$

3. Law of Universal Gravitation

The force of attraction F between two point masses m_1 and m_2 equals:

$$F = \gamma \frac{m_1 m_2}{R^2} \tag{1.33}$$

where R is the distance between the masses, and γ—the gravitational constant equal to 6.67×10^{-8} cm³/g·s² (in the CGS system of units*). The *gravitational constant* is a quantity numerically equal to the force of attraction between two point masses of unit mass separated by unit distance. In the case of homogeneous spheres of masses m_1 and m_2 the force of attraction is given by the same formula, except that R now denotes the distance between the centres of the spheres.

The force of attraction between a body of mass m on the surface of the Earth and the Earth

$$F = \gamma \frac{M_E m}{R_E^2} \tag{1.34}$$

where M_E is the mass of the Earth, and R_E—the radius of the Earth.

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* See p. 260 for the CGS system of units.

All bodies at a given point of the Earth fall with identical accelerations relative to its surface. This acceleration in free fall is usually designated by the symbol g. Because of the daily rotation of the Earth the acceleration g will be due to the vector sum of two forces: the force of attraction to the Earth F [see Eq. (1.34)] and the centripetal force F_c [see Eq. (1.32)]. The resultant of these forces is called the *force of gravity*.

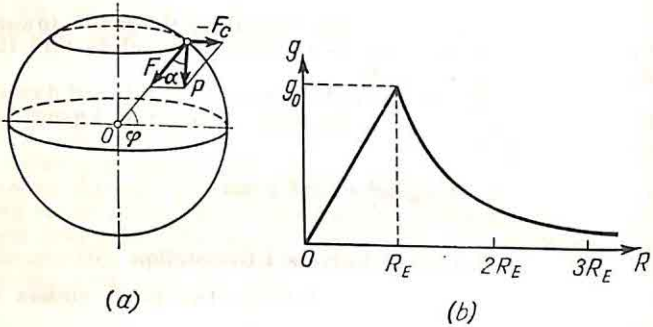


Fig. 6. (*a*) Direction of force of gravity P and force of attraction to the Earth F; (*b*) acceleration of gravity (intensity of gravitational field) versus distance from the centre of the Earth (the Earth is regarded as a homogeneous sphere)

The force of gravity of a body having a mass m is determined by the expression

$$P = mg \tag{1.35}$$

The force of gravity P and the force of attraction to the Earth F differ (insignificantly) from each other in magnitude and direction. The angle α between the directions of the forces P and F (Fig. 6*a*) at the latitude φ is

$$\alpha = 0.0018 \sin 2\varphi$$

The direction of P coincides with that of a plumb line; the force of attraction F is always directed towards the centre of the Earth; the two forces coincide in direction only at the poles, where $P = F$, and at the equator, where $P = F - F_c$.

For this reason and also because the shape of the Earth differs from a spheroidal one, the acceleration of gravity depends on the latitude (see Table 13).

In accordance with the law of gravitation, the acceleration of gravity (the intensity of the gravitational field) at a height H above the surface of the Earth is given by the formula:

$$g=\gamma\frac{M_{\mathrm{E}}}{(R_{\mathrm{E}}+H)^2}$$

or

$$g=g_0\frac{R_{\mathrm{E}}^2}{(R_{\mathrm{E}}+H)^2} \tag{1.36}$$

where g_0 is the acceleration at the Earth's surface.

In the first approximation for $H \ll R_{\mathrm{E}}$

$$g\approx g_0\left(1-2\frac{H}{R_{\mathrm{E}}}\right) \tag{1.37}$$

At the centre of the Earth the intensity of the gravitational field is equal to zero. If the Earth is regarded as a homogeneous sphere, then g increases with increasing distance from the centre of the Earth. Outside the Earth g decreases with increasing distance from the centre of the Earth. The dependence of the acceleration g on the distance R from the centre of the Earth is graphically depicted in Fig. 6*b*.

4. Friction Forces

When two solid bodies in contact are in motion relative to each other, a force arises which hinders this motion. This force is called the *force of friction*. It is caused by the irregularities of the surfaces in contact, as well as by forces of molecular interaction. When there is no layer of liquid between the surfaces, we speak of *dry friction*.

When a body at rest on a flat surface is acted upon by a force parallel to the surface of contact of the bodies, the body will begin to move only when the force reaches a definite magnitude. This magnitude of the force determines the maximum value of *static friction*.

According to the character of the motion giving rise to dry friction we distinguish between *sliding friction* (one body slides over the surface of the other) and *rolling friction* (one body rolls on the surface of the other).

The magnitude of sliding friction F_{fr} depends on the nature and quality of finish of the surfaces in contact and on the force pressing the surfaces together (the normal force F_{n}):

$$F_{\mathrm{fr}} = kF_{\mathrm{n}} \tag{1.38}$$

where k is the *coefficient of friction* depending on the nature and quality of finish of the surfaces in contact, and to a slight degree on the velocity of motion (the dependence on the velocity is usually ignored).

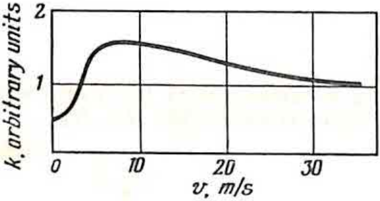


Fig. 7. Dependence of force of friction on the velocity of a steel plate over a steel surface (the quantity k is in arbitrary units)

The coefficient of static friction k_{st} changes its magnitude with a change in the absolute value of the force applied to a body; however, $0 \leqslant k_{\mathrm{st}} \leqslant k$, where k is the coefficient of sliding friction.

Figure 7 shows an approximate dependence of the force of friction on the velocity of a steel plate over a steel surface. The values of the quantity k are given in Table 12.

Rolling friction is less than sliding friction. Rolling friction depends on the radius R of the rolling body, on the normal force, and on the quality of the surfaces in contact:

$$F_{\mathrm{fr}} = k' \frac{F_{\mathrm{n}}}{R} \tag{1.39}$$

where k' is a quantity characterizing the surfaces in contact; k' has the dimension of length.

The following are two examples of the value of k' in cm:

A wheel with a steel tyre on a steel rail	0.05
A cast iron wheel on a steel rail	0.12

5. Density of a Substance

The mass per unit volume of a substance is called its *density* (ρ). The concept of specific gravity is frequently used. *Specific gravity* (d) is the weight of a substance per unit volume:

$$\rho = \frac{m}{V} \tag{1.40}$$

$$d = \frac{P}{V} \tag{1.41}$$

where m is the mass of the body, P—its weight, and V—its volume.

In engineering non-uniform (for example, loose) bodies are characterized by the *volume* (or *bulk*) *density*, which is the mass of a unit volume (1 m^3) of the given substance.

When determining the volume density, the volume is determined with inclusion of the spaces between the particles, lumps, etc. in the relevant material (sand, grain, coal, firewood, etc.).

The units of density are kg/m^3 (SI) and g/cm^3 (CGS); the units of specific gravity—N/m^3 (SI) and dyn/cm^3 (CGS);

$$1\ \mathrm{kg/m^3} = 10^{-3}\ \mathrm{g/cm^3}$$

6. Work, Power, and Energy

Work (A) in physics is the product of the force and the distance through which it acts. If the force does not coincide in direction with the distance, then the work equals:

$$A = Fs \cos \alpha \tag{1.42}$$

where α is the angle between the force (F) and the distance (s) through which the body moves.

The work done when a body moves through the angle φ in rotational motion at a constant moment of force M is

$$A_{\mathrm{rot}} = M\varphi \tag{1.43}$$

Power (N) is the work performed in unit time:

$$N = \frac{A}{t} = Fv \tag{1.44}$$

where F is the force, and v—the velocity.

The power of a rotating body is

$$N_{\mathrm{rot}} = M\omega \tag{1.45}$$

where ω is the angular velocity.

The ability of bodies to perform work is determined by a physical quantity called *energy*. There are two forms of mechanical energy: *kinetic energy* (E_{k}), or the *energy of motion*, which depends on the relative velocity of bodies, and *potential energy* (E_{p}), or the *energy of position*, which depends on the relative position of the bodies. The sum of the kinetic and potential energies of all the bodies in a system is called the *total mechanical energy* of the system.

Forces whose work does not depend on the form of the path are called *conservative* (for example, forces of gravity); forces of friction are not conservative.

If external forces (see p. 33) act on the bodies of a system in addition to the internal conservative forces, the total energy of the system changes.

If we denote by E_1 and E_2 the initial and final energies of the system, then

$$E_2 - E_1 = A$$

where A is the work done by the external forces.

The change in the total energy of a system of bodies between which conservative forces are acting equals the work of the external forces acting on the bodies of the system. In a closed system of bodies (i.e. when external forces are absent and their work $A = 0$) the total energy of the system remains constant. This is one of the fundamental laws of mechanics—the *law of conservation of energy*. The presence in a closed system (see p. 33) of non-conservative forces (for example, forces of friction) results in a reduction in the mechanical energy. In this case the mechanical energy transforms into other kinds of energy (see p. 67).

The kinetic energy of a body equals

$$E_{\mathrm{k}}=\frac{mv^2}{2} \tag{1.46}$$

where m is the mass of the body, and v—its velocity.

The kinetic energy of a rotating body equals

$$E_{\mathrm{k}}=\frac{J\omega^2}{2} \tag{1.47}$$

where J is the moment of inertia, and ω—the angular velocity.

The potential energy of a body in the field of gravitation of the Earth is

$$E_{\mathrm{p}}=-\gamma\frac{M_{\mathrm{E}}m}{R} \tag{1.48}$$

where γ is the gravitational constant (p. 35), M_{E}—the mass of the Earth, m—the mass of the body, and R—the distance from the centre of the Earth to the centre of gravity of the body.

It is customary practice in physics to consider the potential energy of forces of attraction negative, and that of forces of repulsion positive; this is the reason why a minus sign has been used in formula (1.48).

When a body is raised to a small height above the surface of the Earth, the gravitational field of the Earth may be regarded as homogeneous (the acceleration of the free fall is constant in magnitude and direction). In a homogeneous field the potential energy of a body equals

$$E_{\mathrm{p}} = mgh \tag{1.49}$$

where m is the mass of the body, g—the acceleration of free fall, h—the height of the body measured from some arbitrary level at which the value of the potential energy is taken equal to zero. The surface of the Earth can serve, for example, as such an arbitrary level.

The units of work and energy are the joule (SI) and erg (CGS); the units of power—the watt (SI) and erg/s (CGS);

$$1\ \mathrm{J} = 10^7\ \mathrm{erg}$$

TABLES

Table 5

Density of Selected Solids (at 20 °C)

Substance	ρ, 10^3 kg/m^3	Substance	ρ, 10^3 kg/m^3
Metals and alloys		*Wood (air-dry)*	
Aluminium	2.7	Ash, mahogany	0.6-0.8
Bismuth	9.8	Balsa wood	0.2
Brass	8.4-8.7	Bamboo	0.4
Bronze	8.7-8.9	Birch	0.7
Cast iron	7.0	Cedar	0.5-0.6
Chromium	7.15	Ebony	1.1-1.3
Cobalt	8.8	Ironwood (lignum vitae)	1.1-1.4
Constantan	8.88	Oak, beech	0.7-0.9
Copper	8.93	Pine, spruce	0.4-0.5
Duralumin	2.79	Walnut	0.6-0.7
Germanium	5.3		
Gold	19.31	*Minerals and rocks*	
Iron	7.88	Anthracite (dry)	1.2-1.5
Lead	11.35	Apatite	3.16-3.22
Magnesium	1.76	Asbestos	2.35-2.6
Manganin	8.5	Barite	4.48
Molybdenum	10.2	Basalt	2.8-3.2
Nickel	8.9	Bauxite	2.9-3.5
Nickeline	8.77	Beryl	2.67-2.72
Niobium	8.57	Calcite	2.6-2.8
Permalloy	8.6	Chalk (air-dry)	2.0
Permendur	8.2-8.3	Corundum	4.00
Platinum	21.46	Diamond	3.51
Plutonium	19.25	Granite	2.5-3.0
Silicon	2.3	Graphite	2.21-2.25
Silver	10.5	Kaolinite	2.54-2.60
Sodium	0.975	Marble	2.5-2.8
Steel	7.7-7.9	Mica	2.6-3.2
Supermalloy	8.87	Quartz	2.65
Tantalum	16.6		
Thallium	11.86	*Miscellaneous materials*	
Thorium	11.71	Amber	1.1
Tin	7.29	Bakelite varnish	1.4
Titanium	4.5		
Tungsten	19.34		
Uranium	19.1		
Vanadium	6.02		
Zinc	7.15		
Zirconium	6.5		

Table 5 (concluded)

Substance	ρ, 10^3 kg/m^3	Substance	ρ, 10^3 kg/m^3
Beeswax, white	0.95-0.96	*Plastics and laminated plastics*	
Bone	1.8-2.0		
Ebonite	1.2		
Glass, common	2.5	Cellon	1.3
Ditto, mirror	2.55	Fluoroplastics	2.1-2.4
Ditto for thermometers	2.59	Laminated aminoplasts	1.4
Ditto, Pyrex	2.59	Laminated fabric	1.3-1.4
Ditto, quartz	2.21	Phenolic plastic, impregnated	1.34-1.4
Ice (at 0 °C)	0.917	Plexiglas	1.18
Porcelain	2.2-2.4	Polystyrene	1.06
Rubber, ordinary, hard	1.2	Polyvinyl plastic	1.34-1.4
		Vinyl plastic	1.38-1.4

Table 6

Density of Selected Liquids (at 20 °C)

Liquid	ρ, 10^3 kg/m^3	Liquid	ρ, 10^3 kg/m^3
Acetic acid	1-0.49	Hydrochloric acid (38%)	1.19
Acetone	0.791	Liquid petrolatum	0.8
Aniline	1.02	Machine oil	0.9
Benzene	0.879	Mercury	13.55
Bromine	3.12	Methyl alcohol	0.792
Chloroform	1.489	Milk of average fat content	1.03
Crude oil	0.76-0.85	Nitric acid	1.51
Ethyl alcohol	0.79	Nitrobenzene	1.2
Formic acid	1.22	Nitroglycerine	1.6
Gasoline	0.68-0.72	Sea water	1.01-1.03
Glycerine	1.26	Sulphuric acid	1.83
Heavy water (D_2O)	1.1086	Toluene	0.866
Heptane	0.684	Water	0.99823
Hexane	0.660		

Table 7

Density of Selected Metals in the Liquid State

Substance	Temperature, °C	ρ, 10^3 kg/m^3
Aluminium	660	2.380
	900	2.315
	1100	2.261
Bismuth	300	10.03
	600	9.66
	962	9.20
Gold	1100	17.24
	1200	17.12
	1300	17.00
Iron	1530	7.23
Lead	400	10.51
	600	10.27
	1000	9.81
Potassium	64	0.82
Silver	960.5	9.30
	1092	9.20
	1300	9.00
Sodium	100	0.928
	400	0.854
	700	0.780
Tin	409	6.834
	574	6.729
	704	6.640

Table 8

Density of Water and Mercury at Different Temperatures

t, °C	ρ, 10^3 kg/m^3	t, °C	ρ, 10^3 kg/m^3	t, °C	ρ, 10^3 kg/m^3	t, °C	ρ, 10^3 kg/m^3
(a) Density of water							
−10	0.998 15	6	0.999 97	50	0.988 07	250	0.794
−5	0.999 30	7	0.999 93	60	0.988 24	300	0.710
0	0.999 87	8	0.999 88	70	0.977 81	350	0.574
1	0.999 93	9	0.999 81	80	0.971 83	374.15 (critical temperature)*	0.307
2	0.999 97	10	0.999 73	90	0.965 34		
3	0.999 99	20	0.998 23	100	0.958 38		
4	1.000 00	30	0.995 67	150	0.917 3		
5	0.999 99	40	0.992 24	200	0.869 0		
(b) Density of mercury (at pressure of 1 atm)							
0	13.595 1	25	13.533 5	50	13.472 3	75	13.411 6
5	13.582 7	30	13.521 2	55	13.460 1	80	13.399 5
10	13.570 4	35	13.509 0	60	13.448 0	90	13.375 3
15	13.558 0	40	13.496 7	65	13.435 8	100	13.351 4
20	13.545 7	45	13.484 5	70	13.423 7	300	12.875

---

* For critical temperature sec pp. 70, 77.

Table 9

Density of Gases and Vapours (at 0 °C and 760 mm Hg)

Substance	ρ, kg/m³	Substance	ρ, kg/m³
Acetylene	1.173	Ethyl ether (saturated)	0.83
Air	1.293	Helium	0.1785
Ammonia	0.771	Hydrogen	0.08988
Argon	1.783	Krypton	3.74
Benzene (saturated)	0.012	Neon	0.900
Carbon dioxide	1.977	Nitrogen	1.251
Carbon monoxide	1.25	Oxygen	1.429
Chlorine	3.22	Ozone	2.139
Ethyl alcohol (saturated)	0.033	Water vapour (saturated)	0.005

Note. For saturated water vapour see Chap. 2.

Table 10

Volume Density of Selected Substances

Substance	Volume density, kg/m³	Substance	Volume density, kg/m³
Asbestos felt	600	Masonry, red brick	1600-1700
Asbestos paper	850-900	Ditto, silicate brick	1700-1900
Asphalt	2120	Mipora (formaldehyde-urea foam)	not more than 20
Beets	650	Peas	700
Broadcloth	250	Potatoes	670
Cement, powdered	1400	Pressboard (made of reeds)	260-360
Clay, 15-20% moisture content by weight	1600-2000	Reinforced concrete, 8% moisture content by weight	2200
Concrete, dry	1600	Sand	1200-1600
Concrete mixed with crushed rock, 8% moisture content by weight	2000	Sandstone	2600
Corn (grain)	750	Silk	100
Cotton wool, air-dry	80	Slag, blast-furnace	600-800
Foam concrete	300-1200	Ditto, furnace	900-1300
Gravel, air-dry	1840	Slag concrete, 18% moisture content by weight	1500
Hay, compressed	100	Snow, compact	200-400
Ditto, fresh-mown	50	Ditto, fresh-fallen	80-190
Lime plaster, 6-8% moisture content by weight	1100	Woolen cloth	240
Lime, powdered	500	Woolen felt	300

Table 11

Moments of Inertia of Selected Homogeneous Bodies

Body	Axis	J
Thin bar of length l	Perpendicular to bar and passing through its centre	$\frac{ml^2}{12}$
Circular disk or cylinder of radius r	Perpendicular to plane of disk and passing through its centre	$\frac{mr^2}{2}$
Sphere of radius r	Coinciding with diameter	$0.4mr^2$
Thin cylindrical tube or ring of radius r	Coinciding with axis of tube	mr^2
Circular cylinder of length l and radius r	Perpendicular to axis of cylinder and passing through its centre	$m\left(\frac{l^2}{12}+\frac{r^2}{4}\right)$
Rectangular parallelepiped of dimensions $2a$, $2b$, $2c$	Passing through centre and parallel to edge $2a$	$m\frac{b^2+c^2}{3}$

Note. The table gives the moments of inertia of bodies about axes which pass through their centres of gravity. The moment of inertia about an arbitrary axis can be found according to formula (1.30). For example, the moment of inertia of a thin bar about an axis perpendicular to the bar and passing through one of its ends is:

$$J=\frac{ml^2}{12}+m\left(\frac{l}{2}\right)^2=\frac{ml^2}{3}$$

Table 12

Coefficients of Sliding Friction for Selected Materials

Surfaces in contact	k
Bronze on bronze	0.2
Ditto on steel	0.18
Cast iron on bronze	0.21
Ditto on cast iron	0.16
Copper on cast iron	0.27
Dry wood on wood	0.25-0.5
Fluoroplastic-4 on fluoroplastic	0.052-0.086
Fluoroplastic on stainless steel	0.064-0.080
Greased leather belt on metal	0.23
Hempen rope, dry, on oak	0.53
Ditto, wet, on oak	0.33
Ice on ice	0.028
Iron-bound runners on snow and ice	0.02
Leather belt, dry, on metal	0.56
Ditto, moist, on metal	0.36
Ditto, on oak	0.27-0.38
Metal, dry, on oak	0.5-0.6
Ditto, moist, on oak	0.24-0.26
Oak on oak, along grain	0.48
Ditto, along grain of one surface and across grain of other	0.34
Rubber (tyres) on cast iron	0.83
Ditto on hard soil	0.4-0.6
Sliding friction bearing, lubricated	0.02-0.08
Steel on cast iron	0.16
Ditto on ice (skates)	0.02-0.03
Ditto on iron	0.19
Ditto on steel	0.18
Steel (or cast iron) on ferrodo* and Raibest*	0.25-0.45
Steel-rimmed wheel on steel rail	0.16
Wooden runners on snow and ice	0.035

Note. An asterisk denotes materials used in braking and frictional devices.

Table 13

Intensity of Earth's Gravitational Field (Acceleration of Free Fall) for Different Latitudes at Sea Level

Latitude	g, m/s^2	Latitude	g, m/s^2
0°	9.780 30	55°45′ (Moscow)	9.815 23
10°	9.781 86	59°57′ (Leningrad)	9.819 08
20°	9.786 34	60°	9.819 14
30°	9.793 21	70°	9.826 06
40°	9.801 66	80°	9.830 58
50°	9.810 66	90°	9.832 16

Table 14

Dynamic Characteristics of Solar System

Body	D, 10^{12} cm	R, 10^8 cm	ρ, g/cm^3	g, m/s^2	M, 10^{27} g
Sun	—	696	1.41	274	1 984 000
Mercury . .	5.79	2.49	5.13	3.53	0.312
Venus . . .	10.8	6.19	4.97	8.53	4.9
Earth	14.95	6.378	5.52	9.81	5.98
Mars	22.78	3.42	3.94	3.73	0.65
Jupiter . . .	77.8	71.4	1.33	25.9	1901.4
Saturn . . .	142.6	60.4	0.69	11.1	568.8
Uranus . . .	286.8	23.5	1.56	10.5	87.7
Neptune . .	449.4	22.3	2.27	13.83	103
Pluto	589.6	—	—	—	—
Moon	0.038 44 (from Earth)	1.738	3.34	1.62	0.0736

Note. D—distance from the Sun
R—mean radius of a planet
ρ—density of the substance of a planet
g—acceleration of free fall at the surface
M—mass.

C. STATICS OF SOLID BODIES

FUNDAMENTAL CONCEPTS AND LAWS

Statics deals with the conditions of equilibrium of a body or system of bodies. If a number of forces are acting on a body at rest, such that the directions of the forces intersect at a single point, then the body will remain at rest when the vector sum of these forces is zero. A force may be displaced along its line of action.

Centre of gravity of a solid body or system of bodies. Every particle of a body is subjected to the pull of gravity. The point at which the resultant of all the forces of gravity acting on the separate points of a body is applied is called its *centre of gravity*. The sum of the moments of the forces of gravity acting on all the particles of the body relative to its centre of gravity is equal to zero.

Kinds of equilibrium of bodies. When forces appear tending to return a body to its original position after it has been slightly displaced from the position of equilibrium, such an equilibrium is called *stable*.

Generally after the action of small disturbances (displacements, jolts) on a body in stable equilibrium, it begins to oscillate with a small amplitude about the position of equilibrium, these oscillations gradually attenuate owing to friction (see p. 104) and equilibrium is restored.

In a position of stable equilibrium the potential energy of a body has its minimum value (upon the action of conservative forces).

If upon an infinitely small deviation of a body from a position of equilibrium forces appear that tend to increase this deviation, such a position is called *unstable equilibrium*.

In *neutral equilibrium* no forces appear when a body is moved from this position, and the new position is also one of equilibrium.

Conditions of equilibrium of a body on an inclined plane. For a body of weight P to be in equilibrium on an inclined plane which forms an angle α with the horizontal it must be subjected to a force F equal to $F_1 = P \sin \alpha$; the force F must be directed upward along the inclined plane (Fig. 8). The body itself presses on the inclined plane with the force $F_2 = P \cos \alpha$ while the inclined plane acts on the body with an equal force. A body resting freely on an inclined plane will remain at rest as long as the force pulling it down is less than the force of static friction. This condition is satisfied if $\tan \alpha > k$, where k is the coefficient of static friction.

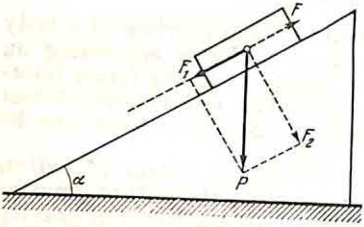


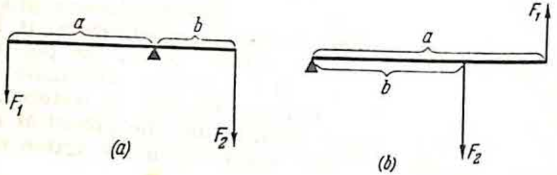
Fig. 8. Equilibrium of a body on an inclined plane

The lever. A lever is in equilibrium if the vector sum of the moments of all the forces applied to it equals zero (Fig. 9):

$$F_1 a - F_2 b = 0$$

where a and b are the lever arms of the applied forces.

The condition that the moments of all the forces are equal also applies to the equilibrium of a windlass (Fig. 10*a*) or a winch.



(*a*) (*b*)

Fig. 9. Levers:
(*a*) fulcrum between forces acting on lever; (*b*) fulcrum at one end of lever

Pulleys. A fixed pulley (Fig. 10*b*) serves only to change the direction of the applied force. A movable pulley (Fig. 10*c*) gives us a gain in force. When a movable pulley is at rest or in uniform rotation, the sum of all the applied forces and the sum of all the moments are equal to zero.

Hence it follows that

$$P = 2F \quad \text{or} \quad F = \frac{P}{2}$$

The pulley block. A pulley block (Fig. 11*a*) is a system of fixed and movable pulleys combined by a common holder.

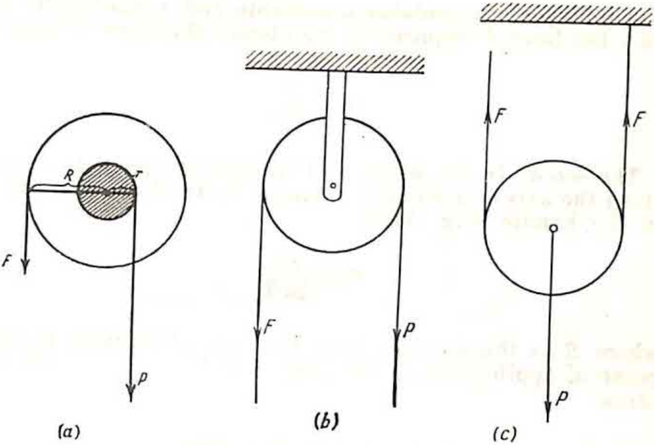


Fig. 10. Schematic diagram of windlass (*a*), fixed pulley (*b*), and movable pulley (*c*)

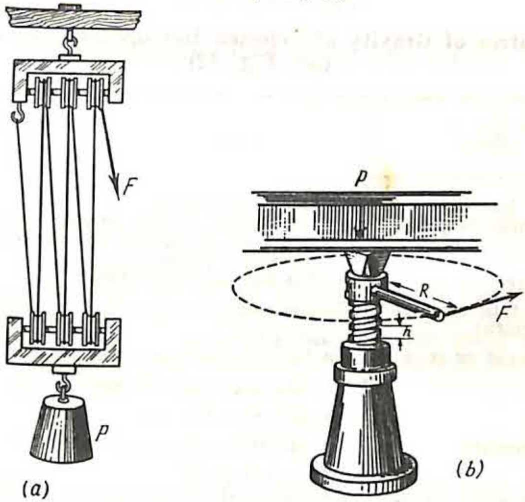


Fig. 11. Pulley block (*a*) and jackscrew (*b*)

If a pulley block contains n movable and n fixed pulleys, then the force F required to counteract the force P equals

$$F=\frac{P}{2n}$$

The screw. In the absence of friction the force P acting along the axis of a screw is balanced by the force F applied to the handle (Fig. 11*b*):

$$F=\frac{Ph}{2\pi R}$$

where R is the distance from the axis of rotation to the point of application of the force, and h—the pitch of the screw.

TABLES AND GRAPHS

Table 15

Centres of Gravity of Selected Homogeneous Bodies (see Fig. 12)

Body	Position of centre of gravity
Thin bar	At the centre of the bar
Cylinder or prism	In the middle of the straight line connecting the centres of the bases of the cylinder or prism
Sphere	At the centre of the sphere
Flat thin circular segment	On the axis of symmetry at $\frac{2}{5}$ its height above the base
Pyramid or cone	On the straight line connecting the centre of the base and the apex at $\frac{1}{4}$ the distance from the base
Hemisphere	On the axis of symmetry at $\frac{3}{8}$ the radius from the centre of the sphere
Thin solid triangular plate	At the point of intersection of the medians

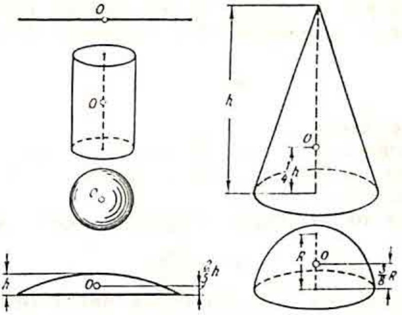


Fig. 12. Position of centre of gravity of selected bodies of regular geometrical shape

D. ELEMENTS OF THE THEORY OF ELASTICITY

FUNDAMENTAL CONCEPTS AND LAWS

Under the action of external forces a solid body undergoes a change in shape, or is deformed. If the body resumes its original shape when the forces are removed, the deformation is called *elastic*.

When a body undergoes elastic deformation, internal elastic forces arise which tend to restore the body to its original shape. The magnitude of these forces is proportional to the deformation.

Deformation by tension and by compression. The increase in length (Δl) of a specimen produced by an external force (F) is proportional to the magnitude of the force and to the original length (l) and is inversely proportional to the cross-sectional area (S):

$$\Delta l=\frac{1}{E}\,\frac{lF}{S} \tag{1.50}$$

where $\frac{1}{E}$ is a coefficient of proportionality. Formula (1.50) expresses *Hooke's law.*

The quantity E is called *Young's modulus* and characterizes the elastic properties of a material. The ratio

$$\frac{F}{S} = p$$

is called the *stress*.

The deformation of rods of arbitrary lengths and cross-sectional areas is described by a quantity called the *longitudinal strain*, $\varepsilon = \Delta l / l$.

For bodies of arbitrary shapes Hooke's law is:

$$p = E\varepsilon \tag{1.51}$$

Young's modulus is numerically equal to the stress required to double the length of a body. Actually, however, a specimen fails at considerably smaller stresses. Figure 13 graphically shows the experimentally determined relation between p and ε, where p_{ult} is the *ultimate stress*—the stress under which a local contraction (neck) appears on the rod, p_{y} is the *yield point*—the stress under which the material begins to flow (the deformation increases without any increase in the applied force), p_{el} is the *elastic limit*—the stress below which Hooke's law is valid.*

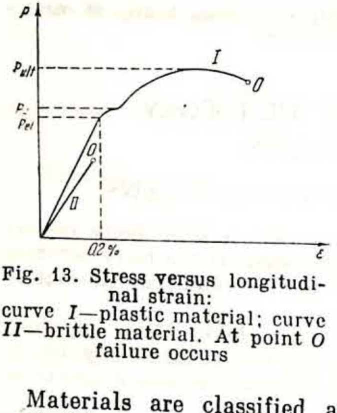


Fig. 13. Stress versus longitudinal strain:
curve *I*—plastic material; curve *II*—brittle material. At point *O* failure occurs

Materials are classified as *brittle* and *plastic*. Brittle materials are destroyed when very small strains are produced in them. Brittle materials can usually withstand greater compression than tension without failing.

Tensile strain is accompanied by a decrease in the diameter of a specimen. If Δd is the change in the diameter, then

$$\varepsilon_1 = \frac{\Delta d}{d}$$

is called the *lateral strain*. Experience shows that $\left|\frac{\varepsilon_1}{\varepsilon}\right| < 1$.

---

* It is assumed that the force is applied for a brief time,

The absolute value of

$$\mu = \left| \frac{\varepsilon_1}{\varepsilon} \right|$$

is called *Poisson's ratio.*

Shear. *Shear* is a deformation in which all the layers of a body parallel to a given plane are displaced relative to one another. In shear the volume of a body remains unchanged. The line segment AA_1 (Fig. 14) equal to the displacement of one plane relative to another is called the *absolute shear.* For small angles of shear the angle $\alpha \cong \tan \alpha = \frac{AA_1}{AD}$ characterizes the relative deformation and is called the *relative* or *unit shear.*

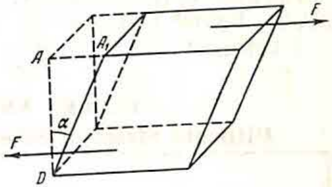


Fig. 14. Shear

Hooke's law for shear can be written in the form:

$$p = G\alpha \tag{1.52}$$

where the coefficient G is called the *shear modulus.*

Compressibility of matter. When a body is subjected to pressure in all directions, its volume decreases by ΔV; as a result elastic forces arise, which tend to restore the body to its original volume. The *compressibility* (β) is defined as the relative change in the volume of a body $\frac{\Delta V}{V}$ produced by a unit change in the stress (p) acting perpendicular to its surface.

The reciprocal of the compressibility is called the *bulk modulus (of elasticity)* (K).

The change in the volume of a body ΔV produced by an increase in pressure Δp in all directions can be computed by the formula:

$$\Delta V = -V\beta\Delta p \tag{1.53}$$

where V is the original volume.

Relation between the constants of elasticity. The Young's modulus, Poisson's ratio, bulk modulus and shear modulus are related to one another by the equations:

$$G = \frac{E}{2(1+\mu)} \quad \text{and} \quad K = \frac{E}{3(1-2\mu)}$$

which permit us, if we know two characteristics of elasticity, to calculate the others in a first approximation.

The potential energy of elastic deformation is given by the formula:

$$E_p = \frac{F\Delta l}{2} \qquad (1.54)$$

where F is the elastic force, and Δl—the deformation.

The units of the moduli of elasticity are N/m^2 (SI), dyn/cm^2 (CGS), kgf/m^2 [MK(force)S] and kgf/mm^2;

$$1\ \text{kgf/mm}^2 = 9.8 \times 10^8\ \text{N/m}^2 = 9.8 \times 10^7\ \text{dyn/cm}^2 = 10^8\ \text{kgf/m}^2$$

TABLES AND GRAPHS

Table 16

Ultimate Stress of Selected Materials (kgf/mm^2)

Material	Ultimate stress	
	in tension	in compression
Aminoplast, laminated	8	20
Bakelite (synthetic resin)	2-3	8-10
Brass, bronze	22-50	—
Brick	—	0.74-3
Cast iron, gray, fine-grained	21-25	up to 140
Ditto, gray, ordinary	14-18	60-100
Ditto, white	—	up to 175
Cellon	4	16
Celluloid	5-7	—
Concrete	—	0.5-3.5
Fluoroplastic-4	2	—
Foam plastic in slabs	0.06	—
Getinax (laminated insulation)	15-17	15-18
Granite	0.3	15-26
Graphite	0.5-1.0	1.6-3.8
Ice (0 °C)	0.1	0.1-0.2
Oak (15% moisture content) across grain	—	1.5
Ditto along grain	9.5	5
Phenolic plastic, impregnated	8-10	10-26
Pine (15% moisture content) across grain	—	0.5
Ditto along grain	8	4
Polyacrylate (organic glass)	5	7
Polystyrene	4	10
Steel, carbon, for machinery	32-80	—
Ditto, for rails	70-80	—
Ditto, silicon-chromium-manganese	155	—
Ditto, structural	38-42	—
Textolite ПТК	10	15-25
Vinyl plastic	4	8

Table 17

Moduli of Elasticity and Poisson's Ratios

Material	Young's modulus E, 10^7 N/m^2	Shear modulus G, 10^7 N/m^2	Poisson's ratio μ
Aluminium	6 300-7 000	2 500-2 600	0.32-0.36
Aluminium bronze, casting	10 300	4 100	0.25*
Bismuth	3 200	1 200*	0.33*
Brass, cold-drawn	8 900-9 700	3 400-3 600	0.32-0.42
Ditto, rolled for shipbuilding	9 800	3 600*	0.36
Cadmium	5 000	1 900*	0.3
Cast iron, white, gray	11 300-11 600	4 400	0.23-0.27
Celluloid	170-190	65*	0.39
Concrete	1 500-4 000	700-1 700	0.1-0.15
Constantan	16 000	6 100	0.33
Copper, casting	8 200	—	—
Ditto, cold-drawn	12 700	4 800	0.33*
Ditto, rolled	10 800	3 900	0.31-0.34
Duralumin, rolled	7 000	2 600	0.31*
Glass	4 900-7 800	1 750-2 900	0.2-0.3
Granite, marble	3 500-5 000	1 400-4 400	0.1-0.15
Invar	13 500	5 500	0.25*
Limestone, dense	3 500	1 500	0.2
Manganin	12 300	4 600	0.33
Nickel	20 400	7 900	0.28*
Phosphor bronze, rolled	11 300	4 100	0.32-0.35
Plexiglas	525	148	0.35*
Quartz filament, fused	7 300	3 100	0.17
Rubber	0.79	0.27	0.46
Ditto, soft, cured	0.15-0.5	0.05-0.15	0.46-0.49
Silver	8 270	3 030	0.37*
Steel, alloyed	20 600	8 000	0.25-0.30
Ditto, carbon	19 500-20 500	8 000	0.24-0.28
Steel casting	17 000	—	—
Titanium	11 600	4 400	0.32*
Wrought iron	15 000	—	—
Zinc, rolled	8 200	3 100	0.27

---

* Calculated values

Table 18

Compressibility of Selected Liquids at Different Temperatures

Substance	Temperature, °C	Pressure range, atm	Compressibility, $\beta \times 10^{-6}$ atm^{-1}
Acetic acid	25	92.5	81.4
Acetone	14.2	9-36	111
	0	100-500	82
	0	500-1000	59
	0	1000-1500	47
	0	1500-2000	40
Benzene	16	8-37	90
	20	99-296	78.7
	20	296-494	67.5
Castor oil	14.8	1-10	47.2
Ethyl alcohol	20	1-50	112
	20	50-100	102
	20	100-200	95
	20	200-300	86
	20	300-400	80
	100	900-1000	73
Glycerine	14.8	1-10	22.1
Kerosene	1	1-15	67.91
	16.1	1-15	76.77
	35.1	1-15	82.83
	52.2	1-15	92.21
	72.1	1-15	100.16
	94	1-15	108.8
Mercury	20	1-10	3.91
Nitrobenzene	25	192	43.10
Olive oil	20.5	1-10	63.3
	14.8	1-10	56.3
Paraffin (melting point 55 °C)	64	20-100	83
	100	20-400	24
	185	20-400	137
Sulphuric acid	0	1-16	302.5
Toluene	10	1-5.25	79
	20	1-2	91.5
Water	20	1-2	46
Xylene	10	1-5.25	74
	100	1-5.25	132

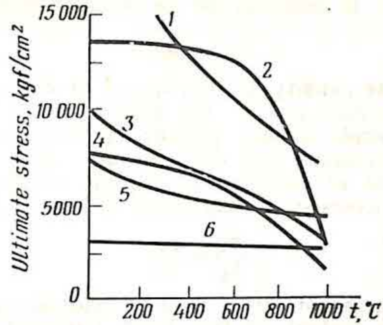


Fig. 15. Temperature dependence of ultimate stress: curve 1—tungsten; 2—nickel steel; 3—cobalt steel; 4—steel N-155; 5—Mo 0.5 Ti; 6—Ti 36 Al

E. MECHANICS OF LIQUIDS AND GASES

FUNDAMENTAL CONCEPTS AND LAWS

Liquids and gases, as distinct from solids, offer no resistance to a change in shape which does not entail a change in volume. To change the volume of a liquid or reduce the volume of a gas one must apply external forces. This property of fluids is called *bulk elasticity*.

Pressure (p) is the perpendicular force acting on a unit surface.

The units of pressure are N/m² (SI), dyn/cm² (CGS), normal atmosphere (atm), and technical atmosphere (at);

$$\begin{aligned} 1\ \mathrm{N/m^2} &= 10\ \mathrm{dyn/cm^2} \\ 1\ \mathrm{atm} &= 1.013 \times 10^5\ \mathrm{N/m^2} \\ 1\ \mathrm{at} &= 9.806\,65 \times 10^4\ \mathrm{N/m^2} \end{aligned}$$

1. Statics

External pressure applied to a gas or liquid is transmitted equally in all directions (*Pascal's law*).

A column of liquid or gas in a uniform gravitational field exerts a pressure caused by the weight of the column. If the liquid or gas is assumed to be incompressible, then the pressure

$$p = \rho g h \tag{1.55}$$

where ρ is the density of the liquid or gas, g—the acceleration of gravity, and h—the height of the column.

The magnitude of the pressure is independent of the shape of the column and depends only on its height.

The heights of columns of liquids in communicating vessels are inversely proportional to their densities:

$$\frac{h_1}{h_2} = \frac{\rho_2}{\rho_1} \tag{1.56}$$

A body immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced (*Archimedes' principle*).

2. Dynamics

When a fluid is in motion with a velocity much smaller than the velocity of sound in that fluid, it may be regarded as incompressible. The motion of fluids gives rise to forces of friction. If these forces are small, they may be ignored, and the liquid or gas is called an *ideal fluid*. Otherwise we have to do with a *viscous fluid*.

Motion of an ideal fluid. A liquid or gas is said to flow at a *steady* rate when the velocity and the pressure remain constant at each point in the stream.

In this case an equal volume of fluid flows through any cross section of a pipe:

$$S_1 v_1 = S_2 v_2 \tag{1.57}$$

where S_1 and S_2 are the areas of two different cross sections of the pipe, and v_1 and v_2—the velocities of the fluid in these cross sections.

When the cross-sectional area of the pipe changes, both the velocity and the pressure of the fluid change in such a manner that in any cross section (for steady flow of an ideal fluid) the following condition is satisfied:

$$\left.\begin{aligned} &p + \rho g h + \frac{\rho v^2}{2} = \text{const} \\ \text{or}\quad &p_1 + \rho g h_1 + \frac{\rho v_1^2}{2} = p_2 + \rho g h_2 + \frac{\rho v_2^2}{2} \end{aligned}\right\} \tag{1.58}$$

where p is the pressure, ρ—the density of the fluid, h—the height of the given cross section of the pipe above a given level, and v—the velocity of the fluid in the given section of the pipe (Fig. 16*a*).

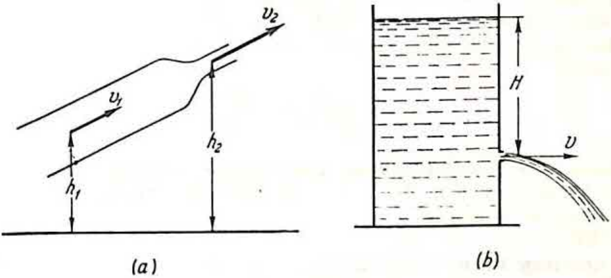


Fig. 16. (*a*) Illustration to formula (1.58); (*b*) outflow of liquid from a small orifice

Equation (1.58) is called *Bernoulli's equation*. From this equation follows *Torricelli's theorem*:

$$v^2 = 2gH \tag{1.59}$$

where v is the velocity of the liquid particles flowing out from a small orifice in the vessel, and H—the height of the surface of the liquid above the orifice (Fig. 16*b*).

Motion of a viscous fluid. When a solid (e.g. a sphere) moves through a fluid, the adjacent layer of fluid adheres to the surface of the solid and moves with it, while the remaining layers of fluid slide over one another. The force acting on a solid moving in a viscous medium (fluid) is opposite in direction to the velocity of the body and is called the *resistance of the medium*. If no eddies are formed in the wake of the moving body, then the resistance of the medium is proportional to the velocity v of the body. In the particular case of a sphere of radius R the resistance of the medium is

$$F = 6\pi\eta R v \tag{1.60}$$

where η is the *coefficient of internal friction* (see p. 73) or the *viscosity*.

The units of the coefficient of internal friction are kg/m·s (SI) and g/cm·s = poise (CGS);

$$1\ \text{kg/m}\cdot\text{s} = 10\ \text{g/cm}\cdot\text{s}$$

Formula (1.60) is called *Stokes' formula.*

When a small sphere falls through a viscous fluid its velocity of uniform (steady) motion is determined by the formula

$$v = g\frac{\rho - \rho_f}{\eta}\frac{2R^2}{9} \qquad (1.61)$$

where ρ is the density of the sphere, R—its radius, ρ_f—the density of the fluid, η—its viscosity, and g—the acceleration of free fall.

The volume of fluid which flows in unit time through a capillary tube of radius R and length l when the pressure difference at the ends of the tube is $p_1 - p_2$ equals

$$V = \frac{1}{\eta}\frac{\pi R^4}{8l}(p_1 - p_2) \qquad (1.62)$$

The viscosity of fluids depends markedly on the temperature.

TABLES

Table 19

Viscosity of Selected Liquids (at 18 °C)

Liquid	η, 10^{-2} kg/m·s	Liquid	η, 10^{-2} kg/m·s
Acetic acid	0.127	Ethyl alcohol	0.122
Acetone	0.0337	Ethyl ether	0.0238
Aniline	0.46	Glycerine	139.3
Benzene	0.0673	Machine oil, heavy	66.0
Bromine	0.102	Ditto, light	11.3
Carbon disulphide	0.0382	Mercury	0.159
Castor oil	120.0	Pentane	0.0244
Chloroform	0.0579	Toluene	0.0613
Cylinder oil, dark	24.0	Water	0.105
Ditto, refined (40 °C)	0.109	Xylene	0.06

Table 20

Viscosity of Selected Gases (at 0 °C)

Gas	η, 10^{-5} kg/m·s	Gas	η, 10^{-5} kg/m·s
Air (without CO_2)	1.72	Hydrogen	0.84
Ammonia	0.93	Methane	1.04
Carbon dioxide . .	1.40	Nitric oxide . . .	1.72
Carbon monoxide	1.67	Nitrogen	1.67
Chlorine	1.29	Nitrous oxide . .	1.38
Helium	1.89	Oxygen	1.92

Table 21

Viscosity of Selected Gases at High Pressures (η, 10^{-6} kg/m·s)

Gas	Temperature, °C	Pressure, atm			
		50	100	600	900
Carbon dioxide	40	18.1	48.3	—	—
Ethylene	40	13.4	28.8	—	—
Nitrogen	25	18 7	19.9	38.7	49.5
	75	20.7	21.7	36.1	44.2

Table 22

Viscosity of Water at Different Temperatures

t, °C	0	5	10	15	20	25	30	40	50
η, 10^{-6} kg/m·s	1797	1518	1307	1140	1004	895	803	655	551

t, °C	60	70	80	90	100	110	120	130	140	150	160
η, 10^{-6} kg/m·s	470	407	357	317	284	256	232	212	196	184	174

Table 23

Viscosity of Selected Liquids at Different Temperatures (η, 10^{-2} kg/m·s)

Liquid \ t, °C	10	20	30	50	70	100
Acetone	0.0358	0.0324	0.0295	0.0251	—	—
Aniline	0.653	0.439	0.318	0.191	0.129	0.076
Benzene	0.076	0.065	0.056	0.0436	0.035	—
Castor oil	244	98.7	45.5	12.9	4.9	—
Transformer oil	4.2	1.98	1.34	0.64	0.38	0.213

Table 24

Viscosity of Air under Different Conditions (η, 10^{-6} kg/m·s)

p, atm \ t, °C	0	25	100
1	17.20	18.37	21.80
20	17.53	18.65	22.02
50	18.15	19.22	22.40
100	19.70	20.60	23.35
200	23.70	23.95	25.30
300	28.60	28.00	28.10

Table 25

Viscosity of Selected Metals in the Liquid State

Metal	t, °C	η, 10^{-3} kg/m·s
Aluminium	700	2.90
	800	1.40
Bismuth	304	1.65
	451	1.28
	600	0.99
Lead	441	2.11
	551	1.69
	844	1.18
Mercury	20	1.54
	50	1.40
	100	1.24
	200	1.03
	300	0.90
Mercury	400	0.83
	500	0.77
	600	0.74
Potassium	100	0.46
	200	0.34
	500	0.185
	700	0.14
Sodium	103.7	0.69
	400	0.25
	700	0.18
Tin	240	1.91
	400	1.38
	600	1.05

CHAPTER II

HEAT AND MOLECULAR PHYSICS

FUNDAMENTAL CONCEPTS AND LAWS

A system of bodies which do not interact at all with surrounding bodies is called an *isolated system*.

An isolated system (for example, a gas in a vessel with non-conducting walls) in the long run passes into a state which no longer changes. This state is called *thermal equilibrium*. The scalar quantity determining the presence of thermal equilibrium between bodies that exchange energy by heat transfer is known as *temperature*. A change in the temperature of a body entails a change in almost all its properties (dimensions, density, elasticity, electrical conductivity, etc.).

The temperature of a body characterizes the kinetic energy of thermal motion of its molecules or atoms (see p. 80).

USSR State Standards provide for two temperature scales—the thermodynamic (Kelvin) and practical (Celsius) scales, the degrees of these scales, which are identical in magnitude, being designated K (thermodynamic) and °C (practical), respectively.

In the thermodynamic scale, where the value of 273.16 K (exactly) is established for the temperature of the triple point of water, the ice melting point at standard pressure is 0.01 K below the triple point. The point that is 273.15 K below the melting point of ice at standard pressure is called *absolute zero*. This point is the beginning of counting temperature according to the Kelvin scale.

The zero point of the Celsius scale is the point at which water is in equilibrium with ice; the point 100 °C on the scale is the boiling point of water at standard pressure.

Other standard points of the practical temperature scale are the boiling points of oxygen (—182.970 °C) and sulphur (444.60 °C), and the melting points of silver (960.8 °C) and gold (1063.0 °C) at standard pressure.

1. Fundamentals of Thermodynamics. Specific Heat

By the *internal energy* of a body (system) is meant the sum of the kinetic energy of chaotic motion of the molecules, the potential energy of their interaction and the intramolecular energy.

Energy can be transferred from one body to another in two ways: the first is by mechanical interaction when work is performed by mechanical or electromagnetic forces (see Chap. IV); the second is by thermal interaction when energy is transferred by the chaotic motion of the molecules at the expense of heat conductivity (see p. 72) or thermal radiation (see p. 212). The quantity of energy transferred in the thermal interaction of bodies is called the *quantity of heat* (or simply the *heat*).

In the SI system heat is measured in joules. It may be measured in special non-system units called calories, however:

$$1\ \mathrm{cal} = 4.1868\ \mathrm{J}$$

The quantity of heat required to raise the temperature of a body of unit mass from t_0 to $t = t_0 + \Delta t$ is denoted by ΔQ. The *mean specific heat* in the given temperature interval $(t - t_0)$ is the ratio $\frac{\Delta Q}{\Delta t}$. The limit of this ratio

$$c_{\mathrm{tr}} = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t}$$

is, by definition, the *true specific heat* at the temperature t_0. The true specific heat depends on the temperature. However, in most cases this dependence is disregarded and it is assumed that the true specific heat (or, simply, the specific heat) can be defined as the quantity of heat required to raise the temperature of a body of unit mass from t °C to $(t + 1)$ °C at any temperature t.

The quantity of heat ΔQ absorbed by a body of mass m when its temperature is increased by Δt equals

$$\Delta Q = cm\Delta t \tag{2.1}$$

where c is the specific heat.

The specific heat of a body depends on the conditions under which it is heated. If the body is heated at constant pressure, then c_p is the *specific heat at constant pressure.* If the volume of the body does not change upon heating, then c_v is the *specific heat at constant volume.* In all cases $c_p > c_v$. The specific heats c_p and c_v for a substance in the solid state differ very little.

The units of the specific heat are J/kg·K (SI), and cal/g·K.

The sum of the quantity of heat ΔQ which a body receives when heated and the work ΔA which it performs during this process equals the change in the internal energy ΔU (the *first law of thermodynamics*):

$$\Delta Q + \Delta A = \Delta U$$

The change in ΔU is determined by the final and initial states and does not depend on the heating process; ΔQ and ΔA do depend on the transition process.

Heat cannot spontaneously pass from a colder body to a hotter one without any other changes in the system (the *second law of thermodynamics*).

The specific heat of a body when its temperature approaches absolute zero tends to zero (the *third law of thermodynamics*).

2. Phase Transitions

By a *phase* is meant a complex of parts of a system having identical properties. For example, ice, water, and water vapour forming a system are different phases of it; graphite and diamond are different phases of a solid (carbon).

The transition from one phase to another is attended by the absorption or liberation of a definite amount of heat, called the *heat of transition.*

The quantity of heat required to melt a body, i.e. to transfer it from the solid to the liquid state, is given by the formula

$$Q = \lambda m \tag{2.2}$$

where m is the mass of the melted body, and λ—the heat of fusion.

The *heat of fusion* is the quantity of heat required to convert a unit mass of a solid at the melting point into a liquid at the same temperature. When a liquid crystallizes (solidifies or freezes), heat is evolved.

The heat of fusion is equal to the heat of crystallization.*

When a substance melts, its volume increases (except for water, gallium, antimony, cast iron, and bismuth, whose volume decreases).

For a given pressure there exists for every substance a definite temperature, called the *melting point*, at which the substance passes from the solid to the liquid state. Throughout this transition the temperature remains constant.

The melting point depends on the pressure. A curve showing how the melting point depends on the pressure is called a *melting curve*. The coefficient of the change in the melting point of water with pressure equals approximately -7.5×10^{-3} K/atm.

The transition of a liquid to the gaseous state is called *evaporation* or *vaporization*; the reverse transition is called *condensation*. The vaporization of solids is called *sublimation*. The molecules having the highest velocity in vaporization escape through the free surface of a body, which cools as a result of this process.

Vaporization which takes place not only on the surface, but within the liquid as well, is called *boiling*. A liquid boils at a definite (for a given external pressure) temperature. This temperature is called the *boiling point*. It remains constant throughout the process of boiling. The coefficient of the change in the boiling point of water with pressure equals approximately $\frac{1}{26.12}$ K/mm Hg.

The quantity of heat which must be imparted to a body to convert it into a vapour, i.e. the *total heat of vaporization*, is

$$Q = rm \tag{2.3}$$

---

* Everything said about temperature and heat of fusion refers to crystalline and polycrystalline bodies. A *crystalline* body is one whose properties are different in different directions. A body composed of numerous chaotically oriented small crystals is called *polycrystalline*.

where m is the mass of the evaporated substance, and r— the *specific heat of vaporization*, equal to the heat needed for the vaporization of a unit mass.

When a vapour condenses, heat is liberated; the heat of vaporization equals the heat of condensation.

The vaporization of a liquid from an open vessel can proceed until all of the liquid is converted into vapour. In a closed vessel vaporization proceeds until a state of equilibrium between the mass of the liquid and that of the vapour is reached. At this stage vaporization and condensation compensate each other. Such a state of equilibrium is called *dynamic*. A vapour which is in dynamic equilibrium with its liquid is called a *saturated vapour*.

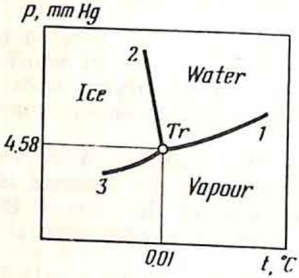


Fig. 17. Curves of vaporization (*1*), melting (*2*), and sublimation (*3*) near the triple point (*Tr*) of water

Boiling occurs at the temperature at which the pressure of the saturated vapour is equal to the external pressure.

The pressure and density of a saturated vapour increase, while the density of a liquid decreases with increasing temperature. The curve showing the temperature dependence of saturated vapour is called the *vaporization* or *saturation curve*; for solid crystalline bodies such curves are called *sublimation curves*.

The melting, vaporization, and sublimation curves determine the conditions of equilibrium of two phases — solid and liquid, liquid and gaseous, solid and gaseous. The point of intersection of all three curves is called the *triple point* (Fig. 17). The latter determines the conditions (pressure, temperature, and density) of the simultaneous equilibrium of three phases.

The curve showing the equilibrium of a liquid and its saturated vapour will continue up to the temperature at which their densities become equal and the meniscus dividing them disappears. This state is called the *critical state*, and the values of the density, pressure, and temperature corresponding to it are called *critical values* (see p. 77).

The specific heat of vaporization depends on the temperature. As the temperature increases, the specific heat of vaporization decreases, and at the critical temperature it becomes zero. The heat of vaporization r is expended on the work performed by the molecules in escaping through the surface layer of the liquid (*internal heat of vaporization* ρ), and the work of expansion upon passing from the liquid to the gaseous state (*external heat of vaporization* ψ). Figure 18 shows the dependence of r, ρ, and ψ on the temperature t for water.

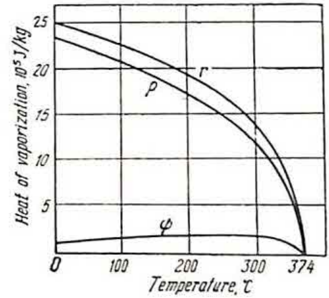


Fig. 18. Temperature dependence of external (φ), internal (ρ), and total (r) heats of vaporization for water

3. Thermal Expansion of Solids and Liquids

A change in the temperature of solids and liquids is accompanied by a change in their linear dimensions and volume. The length of a solid body at a temperature t °C (l_t) is determined by its length at 0 °C (l_0), the temperature (t), and the coefficient of linear expansion (α):

$$l_t = l_0\,(1 + \alpha t) \tag{2.4}$$

The *coefficient of linear expansion* is the mean (for the temperature interval from 0 °C to t °C) increase in unit length of a body for one degree rise in temperature:

$$\alpha = \frac{1}{t}\,\frac{l_t - l_0}{l_0}$$

Similarly, for the volume of a body

$$v_t = v_0\,(1 + \beta t) \tag{2.5}$$

where β is the coefficient of volume expansion.

The *coefficient of volume expansion* is the mean (for a given temperature interval) increase in unit volume of a body for one degree rise in temperature:

$$\beta = \frac{1}{t}\,\frac{v_t - v_0}{v_0}$$

For an isotropic solid (a body whose properties are the same in each direction) $\beta = 3\alpha$.

The coefficients of volume and linear expansion are expressed in K^{-1}.

The following formulas are more exact:

$$\Delta l = l_0\,(at + bt^2) \quad \text{and} \quad l_t = l_0\,(1 + at + bt^2) \quad (2.6)$$

where a and b are coefficients determined experimentally for each substance.

Thus, the coefficient of linear expansion varies with the temperature range in which the body is heated.

For example, for iron $l_t = l_0\,(1 + 117 \times 10^{-7}\,t + 4.7 \times 10^{-9}\,t^2)$, and the coefficient of linear expansion of iron upon heating from 0 to 75 °C equals $1.21 \times 10^{-5}\ K^{-1}$, while for the temperature range 0-750 °C it equals $1.52 \times 10^{-5}\ K^{-1}$.

When a body is heated, its density changes. The density of a body at a temperature t is given by the formula

$$\rho_t = \frac{\rho_0}{1+\beta t} \qquad (2.7)$$

where ρ_0 is the density of the body at 0 °C, and β—the coefficient of volume expansion.

4. Transfer Phenomena

Heat can be transmitted by convection, conduction, and radiation (see thermal radiation, p. 212).

Conduction. *Conduction* is the transfer of heat brought about by the random motions of atoms or molecules.

In fluids temperature differences are evened out mainly by *convection*, by the flow of the fluid from a warmer to a colder region. Convection does not take place in solids.

The quantity of heat transferred through a layer of substance of thickness Δl and cross-sectional area S having a temperature difference ΔT on its planes in the time t is given by

$$Q = \lambda \frac{\Delta T}{\Delta l} St \qquad (2.8)$$

where λ is the thermal conductivity.

The *thermal conductivity* is the quantity of heat transferred in unit time through a layer of unit thickness and unit cross-sectional area when the temperature difference between the two surfaces of the layer is one Kelvin.

Diffusion. *Diffusion* denotes the process of levelling out concentrations due to the transfer of a substance by means of molecular motion. The mass of a substance M transferred through a layer of thickness Δl and cross-sectional area S having a difference in concentrations ΔC on its planes in the time t will be

$$M = D \frac{\Delta C}{\Delta l} St \qquad (2.9)$$

where D is the diffusion coefficient.

The *diffusion coefficient* is a quantity measured by the mass of a diffusing substance transferred through a layer of unit thickness at a difference between the concentrations on its planes of one unit.

The units of the diffusion coefficient are m²/s (SI) and cm²/s (CGS);

$$1 \text{ m}^2/\text{s} = 10^4 \text{ cm}^2/\text{s}$$

The units of the thermal conductivity are W/m·K (SI) and cal/cm·s·K, kcal/m·h·K;

$$1 \text{ W/m}\cdot\text{K} = 2.39 \times 10^{-3} \text{ cal/cm}\cdot\text{s}\cdot\text{K} = 0.86 \text{ kcal/m}\cdot\text{h}\cdot\text{K}$$

Internal friction (viscosity). Upon the relative parallel displacement of layers of a liquid or gas, forces of friction appear in it that retard the motion of the layers travelling at a higher velocity and accelerate the layers having a lower velocity. The cause of viscosity is the transfer of the momentum (impulse) of the orderly motion by molecules that pass from one layer to another.

The magnitude of the force of internal friction is

$$F_{\text{fr}} = \eta \frac{\Delta v}{\Delta l} S \qquad (2.10)$$

where $\frac{\Delta v}{\Delta l}$ is the ratio of the difference between the velocities of the layers to the distance between them, and S— the area of contact of the layers.

The *coefficient of internal friction* η is measured by the force of friction appearing between two layers having a unit area with the ratio $\frac{\Delta v}{\Delta l}$ equal to unity.

The units of viscosity are the poise—g/s·cm (CGS) and kg/s·m (SI).

Relationships (2.8)-(2.10) hold provided that the length of the free path of the liquid or gas molecules (see p. 78) is less than the dimensions of the confining vessel.

5. Surface Tension of Liquids

The molecules in the surface of a liquid experience forces of attraction due to the remaining molecules, which tend to pull them into the liquid.

The surface layer of molecules is in a state resembling that of an elastic film under tension which tends to contract. Every section of the surface layer experiences the pull of all the surrounding sections which keep it in a state of tension. These forces are directed along the surface layer and are called *forces of surface tension.*

The force of surface tension is given by the formula

$$F = \alpha l \qquad (2.11)$$

where l is the perimeter of the surface layer of liquid, and α—the coefficient of surface tension.

The coefficient of surface tension (or, simply, the surface tension) is a quantity numerically equal to the force acting on unit length of a rectilinear boundary of the surface layer of a liquid.

The units of surface tension are N/m (SI) and dyn/cm (CGS);

$$1\ \text{N/m} = 10^3\ \text{dyn/cm}$$

The surface tension decreases with increasing temperature and vanishes at the critical temperature.

6. Gas Laws

The behaviour of most substances in the gaseous state under ordinary conditions is described by the equation

$$pv = \frac{m}{\mu} RT \qquad (2.12)$$

This equation is called the *equation of state of an ideal gas,* or the *Clapeyron-Mendeleev equation.* Here p is the pressure of the gas, v—the volume occupied by m kilograms (grammes) of the gas, μ—a kilomole (gramme-mole), R—the molar gas constant, and T—the temperature according to the thermodynamic scale (in kelvins).

A kilomole (gramme-mole) is the number of kilograms (grammes) of a substance numerically equal to its molecular weight.

The number of molecules in one mole is the same for all substances and is called *Avogadro's constant* N_A:

$$N_A = 6.02 \times 10^{26} \text{ kg-mol}^{-1} = 6.02 \times 10^{23} \text{ g-mol}^{-1}$$

The Clapeyron-Mendeleev equation is valid (in the first approximation) for all substances in the gaseous state if their density is less than that of their saturated vapour at the same temperatures.

From equation (2.12) we deduce Gay-Lussac's law, Charles' law, and Boyle's law. For constant p and m (since R = const and μ is constant for the given substance)

$$v_1 = v_0 \frac{T_1}{T_0}$$

where v_0 and T_0 are the volume and temperature of the gas at 0 °C. Whence follows *Gay-Lussac's law* (the equation of an *isobaric process*):

$$v_t = v_0 \left(1 + \frac{1}{273} t\right) \tag{2.13}$$

where t is the temperature in °C.

For constant v and m we obtain *Charles' law* (the equation of an *isochoric process*):

$$p = p_0 \left(1 + \frac{1}{273} t\right) \tag{2.14}$$

For constant T and m (*isothermal process*) we obtain *Boyle's law*:

$$p_1 v_1 = p_2 v_2 \tag{2.15}$$

The quantity $\alpha = \frac{1}{273.15}$ K^{-1} is called the *coefficient of volume expansion*, or the *coefficient of pressure change* of an ideal gas. For real gases at pressures close to atmospheric or greater, the respective coefficients differ somewhat from this value, The density of a gas ρ may be computed from equation (2.12) if the pressure p, the temperature T, and the molecular weight of the gas are known:

$$\rho=\frac{m}{v}=\frac{\mu p}{RT} \tag{2.16}$$

When a gas expands isothermally (at constant temperature), it performs work against the external pressure. This work is performed mainly at the expense of the heat absorbed from the surroundings. The temperature of the gas and the surroundings remains constant. When the gas is compressed, it releases heat which goes off into the surroundings.

When the volume of a given mass of gas changes without heat entering or leaving the system (*adiabatic process*), the relation between the pressure and the volume is expressed by the equation of an adiabat:

$$pv^{\gamma} = \text{const} \tag{2.17}$$

where $\gamma = \frac{c_p}{c_v}$.

If the density of a gas becomes compatible with that of a saturating vapour at a given temperature, considerable deviations from the equation of state of an ideal gas are observed. In this case account must be taken of the force of interaction between the gas molecules and the volume they occupy. This results in an equation for a real gas. The most widespread use has been found by the *van der Waals equation*:

$$\left[p+\left(\frac{m}{\mu}\right)^2\frac{a}{v^2}\right]\left(v-\frac{m}{\mu}b\right)=\frac{m}{\mu}RT \tag{2.18}$$

where v is the volume occupied by m kilograms (grammes) of a gas, μ—kg-mol (g-mol), a and b—van der Waals constants, which can be determined according to the critical parameters for one mole of the gas—the volume v_{cr}, pressure p_{cr}, and temperature T_{cr} of the substance:

$$a=3p_{cr}v_{cr}^2, \quad b=\frac{1}{3}v_{cr}, \quad R=\frac{8}{3}\frac{v_{cr}p_{cr}}{T_{cr}} \tag{2.19}$$

Van der Waals isotherms are shown in Fig. 19. At temperatures below T_{cr} the isotherms have an S-shaped bend;

at these temperatures three values of the volume correspond to one value of the pressure (for example, the volumes v_1, v_2, and v_3 correspond to the pressure p_1). At temperatures T_{cr} and above it the isotherms have no S-shaped bend.

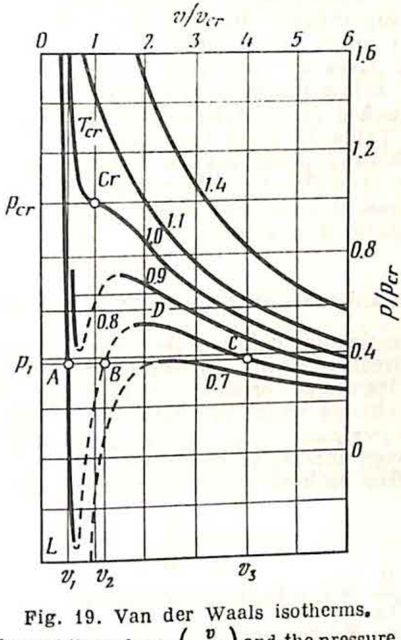


Fig. 19. Van der Waals isotherms.

The relative values of the volume $\left(\frac{v}{v_{\text{cr}}}\right)$ and the pressure $\left(\frac{p}{p_{\text{cr}}}\right)$ are laid off along the axes of abscissas and ordinates, respectively; the figures at the curves show the temperature in relative units $\left(\frac{T}{T_{\text{cr}}}\right)$

The temperature T_{cr} is the *critical temperature* (see also p. 70); the values of the pressure p_{cr} and volume v_{cr} corresponding to it are called the *critical pressure* and the *critical volume*. The state of a substance which T_{cr}, p_{cr}, and v_{cr} correspond to is its critical state.

On the section of the S-shaped bends the isotherms are actually horizontal, i.e. parallel to the axis of abscissas (for the pressure p_1 the isotherm passes through points A, B and C). These sections correspond to equilibrium between the liquid and the gas. A gas (or vapour) in a state of equilibrium with its liquid is a saturated vapour (see p. 70). Under certain conditions an isotherm may pass along sections AL (superheated liquid) and CD (supersaturated vapour), but these states are not stable.

To liquefy gases by increasing their pressure they must be cooled to below the critical temperature. The temperature of a liquefied gas is determined by the pressure which it is under. Table 35 gives the boiling points of liquefied gases. The boiling point can be lowered by lowering the pressure (for example, by exhausting the vapour formed).

In some cases the van der Waals equation also describes the liquid state of a substance.

7. Fundamentals of the Kinetic Theory of Gases

From the molecular point of view a gas consists of a huge number of freely moving particles (molecules or atoms). These particles are in constant motion with different velocities which change when the particles collide.

The *mean free path* is the average path which a molecule travels between successive collisions with other molecules. The mean free path in a gas is given by the formula

$$l = \frac{kT}{\sqrt{2}\,\pi\sigma^2 p} \qquad (2.20)$$

where $k = \frac{R}{N_A}$ is the Boltzmann constant, σ—the diameter of a molecule, T—the absolute temperature (Kelvin scale), N_A—the Avogadro constant, p—the pressure, and R—the molar gas constant.

A law that describes the distribution of molecules by velocities is called a *distribution function*. The distribution function of the molecules of an ideal gas (the *Maxwell distribution*) is given in Fig. 20. The relative number of molecules $\frac{\Delta n}{n}$ having velocities from v to $v + \Delta v$ is laid off along the axis of ordinates, and the values of the velocities are laid off along the other axis. The velocity corresponding to the maximum of the curve in Fig. 20 is called the *most probable velocity* v_{pr}.

The *average velocity* of the molecules is defined by the relation

$$v_{\text{av}} = \frac{v_1 + v_2 + \ldots + v_n}{n} \tag{2.21}$$

where $v_1, v_2, \ldots, v_n$ are the velocities of molecules; the absolute values of the velocities are taken.

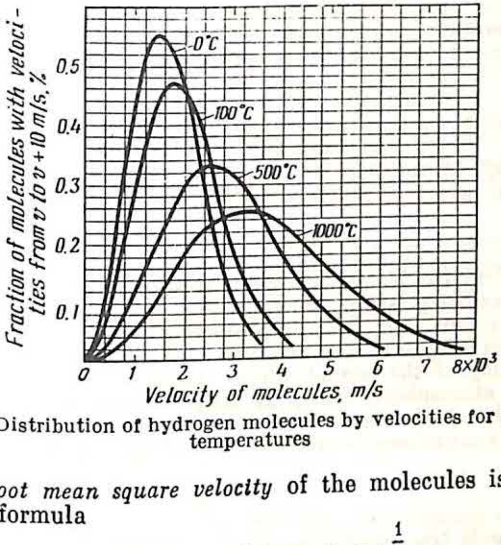


Fig. 20. Distribution of hydrogen molecules by velocities for different temperatures

The *root mean square velocity* of the molecules is given by the formula

$$v_{\text{sq}} = \left(\frac{v_1^2 + v_2^2 + \ldots + v_n^2}{n} \right)^{\frac{1}{2}} \tag{2.22}$$

The following expressions for calculating the velocity are obtained from the Maxwell distribution:

$$v_{\text{pr}}^2 = \frac{2kT}{m}, \quad v_{\text{av}}^2 = \frac{8kT}{\pi m}, \quad v_{\text{sq}}^2 = \frac{3kT}{m} \tag{2.23}$$

where m is the mass of one molecule; here $v_{\text{sq}} > v_{\text{av}} > v_{\text{pr}}$.

The pressure of a gas is due to the impact of individual molecules on the walls of the container. It is equal to

$$p = \frac{1}{3} n m v_{\text{sq}}^2 = nkT \tag{2.24}$$

where n is the number of molecules in unit volume.

The *partial pressure* of a component gas of a mixture is the pressure which the gas would exert if it alone occupied the given volume at the same temperature.

The total pressure of a mixture of ideal gases which do not enter into a chemical reaction is equal to the sum of the partial pressures of the component gases (*Dalton's law*):

$$p = p_1 + p_2 + \dots + p_n \tag{2.25}$$

The average kinetic energy of one molecule of an ideal gas depends only on the temperature:

$$E = \frac{1}{2} ikT \tag{2.26}$$

where $i = 3$ for monoatomic, $i = 5$ for diatomic, and $i = 6$ for polyatomic gases. The kinetic energy of one mole of an ideal gas

$$E_{\mu} = \frac{1}{2} iRT \tag{2.27}$$

Some molecules whose velocities are greater than the escape velocity (see p. 28) can escape from the upper layers of the atmosphere into interplanetary space.

The atmosphere is a mixture of gases held by the field of gravity of the planet which it surrounds. The pressure of the atmosphere decreases with increasing distance (h) from planet's surface. If we assume that the temperature of the atmosphere is independent of the altitude, then

$$p = p_0 e^{-\frac{\mu g h}{RT}} \tag{2.28}$$

where μ is the average molecular weight of the mixture of gases comprising the atmosphere, g—the acceleration of free fall near the surface of the planet, R—the molar gas constant, T—the absolute temperature, p_0—the pressure of the atmosphere at the surface of the planet, and e—the base of natural logarithms ($e \approx 2.72$). Equation (2.28) is called the *barometric formula*.

For the Earth this formula can be written in the form

$$h = 8000 \log \frac{p_0}{p}$$

where h is the altitude in metres.

In many countries, including the USSR, scientific data are often corrected for a standard atmospheric pressure, which is taken equal to the pressure at sea level at 15 °C when the mercury barometer registers 760 mm and the temperature falls by 6.5 K with every 1000 m increase in altitude.

Table 54 gives the relation between the altitude, pressure, density, and temperature of a standard atmosphere.

The air about us always contains a certain amount of water vapour. The mass of water vapour contained in 1 cubic metre of air is called the *absolute humidity*. The absolute humidity can be measured by the partial pressure of the water vapour.

As the absolute humidity increases, the water vapour approaches the state of saturation. The *maximum absolute humidity** at a given temperature is the mass of saturated water vapour contained in 1 cubic metre of air.

The *relative humidity* is the ratio of the absolute humidity to the maximum absolute humidity at a given temperature, expressed in per cent.

The thermal conductivity, coefficient of internal friction (viscosity), and the diffusion coefficient for a gas (λ, η, and D) are calculated by the formulas

$$\lambda=\frac{1}{3}\rho v_{\text{av}} l c_v \tag{2.29}$$

$$\eta=\frac{1}{3}\rho v_{\text{av}} l \tag{2.30}$$

$$D=\frac{1}{3} v_{\text{av}} l \tag{2.31}$$

where ρ is the density of the gas, v_{av}—the average velocity of the gas molecules, c_v—the specific heat at constant volume, and l—the mean free path.

If the mean free path is greater than the dimensions of the vessel or container, internal friction is absent, and the coefficient of friction against the walls of the vessel during motion of the gas is

$$\eta=\frac{1}{6}\rho v_{\text{av}} \tag{2.32}$$

The thermal conductivity in this case is

$$\lambda=\frac{1}{6}\rho v_{\text{av}} c_v \tag{2.33}$$

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* Under certain conditions supersaturation of the vapour can set in.

TABLES AND GRAPHS

Table 26

Specific Heat Capacity, Heat of Fusion, Heat of Vaporization, Melting and Boiling Points for Selected Substances

Substance	c_p, 10^3 J/kg·K at 20 °C	t_m, °C	λ, 10^3 J/kg	t_b, °C	r, 10^3 J/kg
Acetone	2.18	−94.3	96	56.2	524
Aluminium	0.88	658.7	322-394	2300	9220
Benzene	1.705	5.5	127	80.2	396
Bismuth	0.13	271.3	50	1560	855
Brass	0.38	900	—	—	—
Carbon disulphide	1.006	−112	66.6	46.2	348
Cast iron	0.50	1100-1200	96-138	—	—
Copper	0.39	1083	214	2360	5410
Ethyl alcohol	2.43	−114	105	78.3	846
Ethyl ether	2.35	−116.3	113	34.6	351
Fluoroplastic-4	0.92-1.05	—	—	—	—
Germanium	0.31	958	478	2700	—
Glycerine	2.4	—	176	290	825
Gold	0.13	1063	66.6	2800	1575
Ice (water)	4.19	0	334	100	2260
Iron	0.45	1530	293	3050	6300
Lead	0.13	327.3	22.5	1750	880
Lithium	4.40	186	628	1317	20500
Magnesium	1.3	651	373	1103	5450
Mercury	0.138	−38.9	11.73	356.7	285
Naphthalene	1.3	80.3	151	218	316
Nickel	0.46	1452	243-306	3000	7210
Potassium	0.763	64	60.8	760	2080
Silver	0.235	960.8	88	2160	2350
Sodium	1.3	98	113	883	4220
Steel	0.46	1300-1400	205	—	—
Tin	0.23	231.9	59	2270	3020
Toluene	1.73	−95.1	72.1	110.7	365
Wood:					
oak, 6-8% moisture content by weight	2.4	—	—	—	—
pine, 8% moisture content by weight	1.7	—	—	—	—
Wood's alloy	0.17	65.5	35	—	—

Note. c_p—specific heat capacity
λ—heat of fusion
r—heat of vaporization
t_m and t_b—melting and boiling points.

Table 27

Relative Change in Volume of a Substance upon Melting

Substance	$\frac{\Delta v}{v}$, %	Substance	$\frac{\Delta v}{v}$, %
Aluminium	6.6	Ice (water)	−8.3
Aluminium alloys	4.5-5.9	Indium	2.5
Antimony	−0.94	Lead	3.6
Bismuth	−3.32	Lithium	1.5
Cadmium	4.74	Magnesium	4.2
Carbon steel	4.5-6.0	Mercury	3.6
Cesium	2.6	Potassium	2.41
Copper alloys	3.0-4.5	Silver	4.99
Gallium	−3	Sodium	2.5
Gold	5.19	Tin	2.6
Gray iron	2.4-3.6	Zinc	6.9

Table 28

Melting Point of Refractory Materials

Material	t, °C	Material	t, °C
Tantalum and zirconium carbides	3500-3900	Tantalum	2950
Tungsten	3416	Niobium	2415
Zirconium and hafnium borides	3000-3200	Zirconium	1860
		Titanium	1725

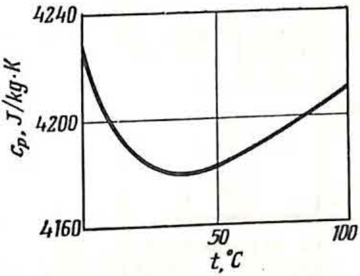

Fig. 21. Specific heat capacity of water at different temperatures

Table 29

Specific Heat Capacity of Solids at Low Temperatures (J/kg·K)

Substance	Temperature, K							
	20 (H_2 boils)	50	77 (N_2 boils)	90 (O_2 boils)	100	150	200	298
Aluminium	10.3	144	349	426	485	686	800	900
Copper . . .	7.9	9.8	202	237	260	331	366	396
Fluoroplastic-4 . . .	77.6	210	316	364	399	553	695	1120
Iron	4.6	54	147	189	221	332	393	447
Nickel . . .	5.0	68.6	168	209	238	336	392	445
Quartz, fused	25.7	115	201	244	274	420	540	740
Steel (stainless) . . .	4.6	67	163	214	244	364	424	477

Table 30

Specific Heat Capacity of Liquid Ethyl Alcohol at Different Temperatures and Pressures (c_p, 10^3 J/kg·K)

Temperature, °C Pressure, kgf/cm^2	−60	−40	−20	0	20	40	60
10	1.59	1.79	1.99	2.20	2.41	2.62	2.84
60	1.59	1.78	1.98	2.17	2.38	2.58	2.79

Temperature, °C Pressure, kgf/cm^2	80	100	120	140	160	180	200
10	3.06	3.28	3.52	3.75	—	—	—
60	3.00	3.21	3.44	3.66	3.90	4.19	4.57

Table 31

Specific Heat Capacity of Gases (c_p, 10^3 J/kg·K at pressure of 1 atm)

Temperature, °C	Oxygen		Air		Carbon dioxide		Water vapour		Ethyl alcohol	
	c_p	$\frac{c_p}{c_v}$	c_p	$\frac{c_p}{c_v}$	c_p	$\frac{c_p}{c_v}$	c_p	$\frac{c_p}{c_v}$	c_p	$\frac{c_p}{c_v}$
0	0.9149	1.397	1.006	1.400	0.8148	1.301	—	—	1.341	1.16
100	0.934	1.385	1.010	1.397	0.9136	1.260	1.103	1.28	1.689	1.12
200	0.964	1.37	1.027	1.390	0.9927	1.235	1.978	1.30	2.011	1.10
300	0.9948	1.353	1.048	1.378	1.057	1.217	2.015	1.29	2.321	1.08
600	1.069	1.321	1.115	1.345	1.192	1.188	2.208	1.26	3.168	1.06

Table 32

Heat of Vaporization

Substance	At temperature, °C	r, 10^3 J/kg
Air (20% O_2)	—	213
Chloroform	61.2	247
Freon-11 ($CFCl_3$)	0	189
Freon-12 ($CFCl_2$)	0	155
Glycerine	100	828
Kerosene	160-230	210-230
Naphthalene	220	316
Nitric acid	—	482
Petrol	50-120	230-314
Sulphuric acid	—	512

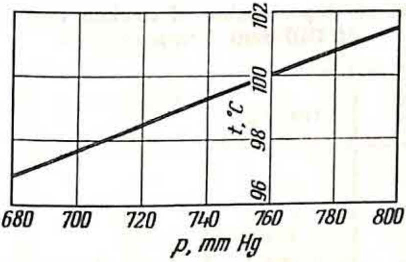

Fig. 22. Boiling point of ordinary water depending on atmospheric pressure

Table 33

Heat of Vaporization at Different Temperatures (r, 10^4 J/kg)

t, °C	Alcohols			Ethyl ether	Acetic acid	Benzene
	methyl	ethyl	propyl			
0	122	92.7	—	38.8	—	—
20	119	92.5	—	36.7	35.2	—
40	116	92.0	—	34.7	36.5	—
60	113	89.4	—	32.9	37.6	—
80	109	86.6	72.6	30.8	38.4	40.1
100	103	82.7	68.8	28.7	38.7	38.3
120	97.4	77.3	64.2	26.1	39.6	36.3
140	90.6	71.7	59.8	23.4	38.5	34.7
160	83.1	65.8	54.1	19.3	37.6	33.1
180	74.3	58.4	48.8	13.4	36.8	31.3
200	68.8	48.7	42.9	—	35.8	28.8
220	47.2	37.0	35.8	—	34.4	26.1
240	—	16.9	26.6	—	32.8	22.7
260	—	—	14.1	—	30.3	18.4
280	—	—	—	—	26.6	11.5

Table 34

Heat of Vaporization of Carbon Dioxide at Different Temperatures

Temperature, °C	r, 10^3 J/kg	Temperature, °C	r, 10^3 J/kg
−50	338	0	237
−40	320	20	155
−30	304	30	63
−10	262	31.1	0.0

Table 35

Melting Point T_m at Triple Point, Heat of Fusion λ, Boiling Point T_b at Standard Pressure, and Heat of Vaporization r of Liquefied Gases

Liquefied gas	T_m, K	λ, J/g·mol	T_b, K	r, J/g·mol
Air	60	—	81	6080
Argon	83.8	1180	87.3	6610
Carbon dioxide	216.4 (at 5 atm)	7950	194.7 (sublimation)	16 500
Fluorine	55.2	1520	85.2	6460
Helium[1]	—	14	4.2	93.8
Hydrogen	14.0	117	20.4	944
Neon	24.6	366	27.1	1770
Nitrogen	63.2	713	77.3	5530
Oxygen	54.4	445	90.2	6840

Note. The heat of fusion corresponds to the melting point at the triple point, and the heat of vaporization to the boiling point at standard pressure.

Table 36

Density, Freezing Point, and Boiling Point of Aqueous Solution of Sodium Chloride at Different Concentrations and Standard Pressure

Density of solution at 15 °C, ρ, 10^3 kg/m^3	Content of NaCl, kg per 100 kg of water	Freezing point, °C	Boiling point, °C
1.01	1.5	−0.9	100.2
1.02	3.0	−1.8	100.4
1.03	4.5	−2.6	100.6
1.04	5.9	−3.5	100.8
1.05	7.5	−4.4	101.2
1.06	9.0	−5.4	101.4
1.07	10.6	−6.4	101.7
1.08	12.3	−7.5	102.0
1.09	14.0	−8.6	102.3
1.10	15.7	−9.8	102.7
1.11	17.5	−11.0	103.1
1.12	19.3	−12.2	103.5
1.13	21.2	−13.6	103.9
1.14	23.1	−15.1	104.4
1.15	25.0	−16.0	104.9
1.16	26.9	−18.2	105.4
1.17	29.0	−20.0	105.9
1.75	30.1	−21.2	106.2

Table 37

Maximum Boiling Points of Aqueous Solutions of Selected Salts at Standard Pressure

Substance	Concentration of substance, kg per 100 kg of water at boiling point	t, °C
$Ba(NO_3)_2$	27.5	101.7
$CaCl_2$	305	178
$CuSO_4$	82.2	104.2
KI	220	185
LiCl	151	168
NaCl	40.7	108.8
$NaNO_3$	222	120

Note. Concentrations are given at which the boiling points of the solutions are maximum.

Table 38

Properties of Ordinary and Heavy Water

	t_m, °C	Temperature of maximum density, °C	t_b, °C	Critical temperature, °C	Critical pressure, atm	Density ρ, 10^3 kg/m^3	
						in critical state	maximum
Water	0	3.98	100	374.15	225.65	0.315	1
Heavy water	3.82	11.23	101.43	371.5	218	0.363	1.106

Table 39

Critical Parameters

Substance	t_{cr}, °C	p_{cr}, atm	ρ_{cr}, 10^3 kg/m^3
Acetic acid	321.6	57.2	0.351
Acetone	235	47	0.268
Benzene	288.6	47.7	0.304
Carbon dioxide	31.1	73	0.460
Ethyl alcohol	243.1	63.1	0.276
Helium	−267.9	2.26	0.069
Hydrogen	−239.9	12.8	0.031
Methane	−82.5	45.8	0.162
Methyl alcohol	240	78.7	0.272
Naphthalene	468.2	39.2	—
Nitrogen	−147.1	33.5	0.311
Oxygen	−118.8	49.7	0.430
Propyl alcohol	263.7	49.95	0.273
Toluene	320.6	41.6	0.292
Water	374.15	225.65	0.315

Table 40

Temperature and Pressure for Triple Points of Selected Substances

Substance	T, K	p, 10^5 N/m^2	Substance	T, K	p, 10^5 N/m^2
Ammonia	195.5	0.0606	Oxygen	54.33	0.00152
Carbon dioxide	216	5.18	Parahydrogen	13.81	0.0704
Neon	24.56	—	Water	273.16	0.0061
Nitrogen	63.15	0.1253			

Table 41

Properties of Saturated Water Vapour

Pressure, kgf/cm^2	Temperature, °C	Specific volume, m^3/kg	Heat of vaporization r, 10^3 J/kg
0.006	0	207	2500
0.02	17.2	63.3	2457
0.1	45.4	14.96	2388
0.2	59.7	7.8	2360
0.4	75.4	4.071	2322

Table 41 (concluded)

Pressure, kgf/cm^2	Temperature, °C	Specific volume, m^3/kg	Heat of vaporization r, 10^3 J/kg
0.6	85.45	2.785	2297
0.8	93.0	2.127	2278
0.9	96.2	1.905	2269
1	99.1	1.726	2262
1.0333	100	1.674	2260
1.23	105	1.42	2242
1.8	116.3	0.996	2215
2	119.6	0.902	2206
3	132.9	0.617	2168
4	142.9	0.4708	2137
5	151.1	0.3818	2111
6	158.1	0.3214	2088
7	164.2	0.2778	2067
8	169.6	0.2448	2048
9	174.5	0.2189	2031
10	179.0	0.1980	2014
12	187.1	0.1663	1984
14	194.1	0.1434	1956
16	200.4	0.1261	1930
18	206.2	0.1125	1907
20	211.4	0.1015	1882
30	232.8	0.0679	1790
40	249.2	0.0506	1712
56.1	270	0.0356	1605
75.9	290	0.0255	1480
101	310	0.0183	1320
131	330	0.0130	1140
169	350	0.00881	893
215	370	0.00493	440
225.2	374	0.00347	113
225.65	374.15	0.00326	0

Table 42

Coefficient of Volume Expansion of Liquids (for Temperatures of about 18 °C)

Substance	β, 10^{-4} K^{-1}	Substance	β, 10^{-4} K^{-1}
Acetone	14.3	Nitric acid	12.4
Aniline	8.5	Petroleum	9.2
Benzene	10.6	Propyl alcohol	9.8
Carbon disulphide	11.9	Toluene	10.8
Chloroform	12.8	Turpentine	9.40
Ethyl alcohol	11.0	Water at 5-10 °C	0.53
Ethyl ether	16.3	Ditto 10-20 °C	1.50
Glycerine	5.0	Ditto 20-40 °C	3.02
Kerosene	10.0	Ditto 40-60 °C	4.58
Mercury	1.8	Ditto 60-80 °C	5.87
Methyl alcohol	11.9		

Table 43

Coefficient of Linear Expansion of Solids (for Temperatures of about 20 °C)

Substance	α, 10^{-6} K^{-1}	Substance	α, 10^{-6} K^{-1}
Aluminium	22.9	Iridium	6.5
Bismuth	13.4	Iron, cast	10.2
Brass	18.9	Ditto, pig	10.4
Brick masonry	5.5	Ditto, wrought	11.9
Bronze	17.5	Lead	28.3
Carbon (graphite)	7.9	Magnesium	25.1
Cement and concrete	12.0	Nickel	13.4
Constantan	17.0	Platinum	8.9
Copper	16.7	Platinum-iridium alloy	8.7
Diamond	0.91	Porcelain	3.0
Duralumin	22.6	Quartz (fused)	0.5
Ebonite	70	Steel 3 (grade 20)	11.9
German silver	18.4	Ditto, stainless	11.0
Glass, ordinary	8.5	Tin	21.4
Ditto, Pyrex	3	Tungsten	4.3
Gold	14.5	Vinyl plastic	70
Granite	8.3	Wood, across grain	50-60
Ice from −10 °C to 0 °C	50.7	Ditto, along grain	2-6
Invar (36.1% Ni)	0.9	Zinc	30.0

Table 44

Coefficient of Linear Expansion at Different Temperatures (α, 10^{-6} K^{-1})

Substance	Temperature, K				
	0	40	100	200	300
Aluminium	0	1	11	19.5	23
Copper	0	1	9.5	15	17.5
Fluoroplastic-4	0	35	55	95	282
Glass (Pyrex)	0	−0.5	1.6	2.5	3.2
Steel, low-carbon	0	0.5	5	10	11.5
Ditto, stainless	0	−0.2	8	13.5	16
Titanium	0	0.5	4	7	8.5

Table 45

Surface Tension of Liquids
(at 20 °C)

Substance	α, 10^{-3} N/m	Substance	α, 10^{-3} N/m
Acetic acid	27.8	Methyl alcohol	22.6
Acetone	23.7	Nitric acid	59.4
Aniline	42.9	Nitrobenzene	43.9
Benzene	29.0	Olive oil	33.06 (18 °C)
Castor oil	36.4 (18 °C)	Petroleum	26
Ethyl alcohol	22.8	Propyl alcohol	23.8
Ethyl ether	16.9	Sulphuric acid, 85%	57.4
Glycerine	59.4	Toluene	28.5
Kerosene	28.9 (0 °C)	Water	72.8

Table 46

Surface Tension of Water and Ethyl Alcohol at Different Temperatures
(α, 10^{-3} N/m)

Temperature, °C / Substance	0	30	60	90	120	150
Ethyl alcohol	24.4	21.9	19.2	16.4	13.4	10.1
Water	75.6	71.18	66.18	60.75	54.9	48.63

Temperature, °C / Substance	180	210	240	300	370
Ethyl alcohol	6.7	3.3	0.1	—	—
Water	42.25	35.4	28.57	14.40	0.47

Table 47

Surface Tension of Metals in the Liquid State

Metal	Temperature, °C	α, 10^{-3} N/m
Aluminium	750	520
Bismuth	300	376
	400	370
	500	363
Lead	350	442
	450	438
	500	431
Mercury	20	465
	112	454
	200	436
	300	405
	354	394
Potassium (in atm of CO_2)	64	410
Sodium	100	206.4
	250	199.5
Tin	300	526
	400	518
	500	510

Table 48

Thermal Conductivity of Selected Materials

Substance	Moisture content, % of weight	λ, W/m·K
Metals		
Aluminium	—	209.3
Brass	—	85.5
Cast iron	—	62.8
Copper	—	389.6
Gold	—	312.8

Table 48 (concluded)

Substance	Moisture content, % of weight	λ, W/m·K
Iron	—	74.4
Mercury	—	29.1
Silver	—	418.7
Steel	—	45.4
Thermal-insulating materials		
Asbestos cardboard	Air-dry	0.157
Asbestos felt	Ditto	0.052-0.093
Asbestos paper	Ditto	0.077-0.134
Foam concrete	Ditto	0.07-0.32
Foam glass	Ditto	0.073-0.107
Foam plastics	Ditto	0.043-0.058
Furnace slag	Ditto	0.233-0.372
Glass wool	—	0.035-0.081
Mipora (formaldehyde-urea foam)	—	0.038
Peat slabs	—	0.047-0.07
Pressboard (made of reeds)	Air-dry	0.105
Wool felt	Ditto	0.047-0.058
Miscellaneous materials		
Brick masonry	Air-dry	0.67-0.87
Cardboard	Ditto	0.14-0.35
Clay	15-20	0.7-0.93
Concrete with crushed rock	8	1.28
Corkboard	0	0.042-0.054
Fluoroplastic-3	—	0.058
Fluoroplastic-4	—	0.233
Glass (ordinary)	—	0.74
Granite	—	3.14
Gravel	Air-dry	0.36
Ice	—	2.21
Leather	Air-dry	0.14-0.16
Oak, across grain	6-8	0.2-0.21
Ditto, along grain	6-8	0.35-0.43
Paper, ordinary	Air-dry	0.14
Phenolic resin lacquer	—	0.29
Pine, across grain	8	0.14-0.16
Ditto, along grain	8	0.35-0.41
Plaster	6-8	0.791
Reinforced concrete	8	1.55
Slag concrete	13	0.698
Snow beginning to thaw	—	0.64
Ditto, compact	—	0.35
Ditto, freshly fallen	—	0.105
Vinyl plastic	—	0.126

Table 49

Thermal Conductivity of Asbestos at Different Temperatures
($\rho = 576$ kg/m^3)

t, °C	0	50	100	150
λ, W/m·K	0.15	0.18	0.195	0.205

Table 50

Thermal Conductivity of Foam Concrete at Different Temperatures
($\rho = 400$ kg/m^3)

t, °C	−18	65	90	126	160
λ, W/m·K	0.11	0.11	0.125	0.17	0.175

Table 51

Thermal Conductivity of Liquids at Different Temperatures (Along Saturation Line, W/m·K)

Substance	Temperature, °C		
	0	50	100
Acetone	0.17	0.16	0.15
Aniline	0.19	0.177	0.167
Benzene	—	0.138	0.126
Castor oil	0.184	0.177	0.172
Ethyl alcohol	0.188	0.177	—
Glycerine	—	0.283	0.288
Methyl alcohol	0.214	0.207	—
Toluene	0.142	0.129	0.119
Vaseline oil	0.126	0.122	0.119
Water	0.551	0.648	0.683

Table 52

Thermal Conductivity of Gases at Standard Pressure

Substance	Temperature, °C	λ, 10^{-4} W/m·K
Air	20	257
Argon	41	187
Carbon dioxide	20	162
Helium	43	1558
Hydrogen	15	1754
Methane	0	307
Nitrogen	15	251
Oxygen	20	262

Table 53

Thermal Coefficient of Pressure Change for Selected Gases

Gas	Air without CO_2	Ammonia	Carbon dioxide	Helium	Hydrogen	Nitrogen	Oxygen
α, 10^{-3} K^{-1}	3.674	3.802	3.726	3.660	3.662	3.674	3.674

Table 54

Standard Atmosphere

Altitude, m	Pressure, $\frac{p}{p_0}$	Density, $\frac{\rho}{\rho_0}$	Temperature, °C
0	1	1	15
1000	0.887	0.907	8.5
2000	0.784	0.822	2
3000	0.692	0.742	−4.5
4000	0.608	0.669	−11
5000	0.533	0.601	−17.5
6000	0.465	0.538	−24
7000	0.405	0.481	−30.5
8000	0.351	0.428	−37
9000	0.303	0.381	−43
10 000	0.261	0.337	−50

Note. p_0 and ρ_0—pressure and density of air in standard conditions.

Table 55

Coefficient of Diffusion of Gases and Vapours in Air (at 0 °C and 760 mm Hg)

Gas	D, 10^{-4} m²/s	Gas	D, 10^{-4} m²/s
Acetic acid	0.107	Hydrogen	0.64
Acetylene	0.19	Methane	0.2
Ammonia	0.2	Methyl alcohol . .	0.13
Benzene	0.078	Oxygen	0.18
Carbon dioxide . .	0.14	Petrol, aviation .	0.079
Carbon disulphide	0.09	Toluene	0.07
Ethyl alcohol . . .	0.10	Water vapour . .	0.21
Ethyl ether	0.08		

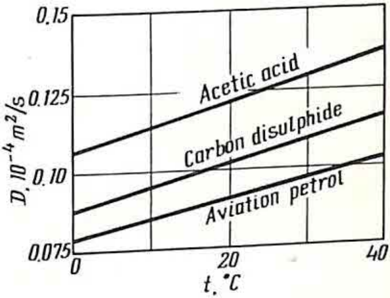

Fig. 23. Temperature dependence of coefficient of diffusion of gases and vapours in air

Table 56

Gas Kinetic Diameters of Molecules

Substance	Diameter d, 10^{-10} m	Substance	Diameter d, 10^{-10} m
Argon	3.6	Mercury	3.0
Carbon dioxide . .	4.54	Methane	4.44
Carbon monoxide .	3.70	Neon	3.54
Chlorine	5.44	Nitrogen	3.7
Helium	2.15	Oxygen	3.56
Hydrogen	2.7	Xenon	4.0
Krypton	3.14		

Table 57

Coefficient of Diffusion of Aqueous Solutions in Pure Water

Substance	t, °C	Concentration of solution	D, 10^{-9} m^2/s
Ammonia	12	1.0	1.64
	4	3.55	1.23
		3.75	0.52
Calcium chloride	10	0.27	0.79
		2.0	0.79
Cane sugar	18.5	0.30	0.36
		0.97	0.28
		1.97	0.50
Copper sulphate	17	0.10	0.45
		0.50	0.34
		1.95	0.27
Ethyl alcohol	11	0.05	0.84
		0.25	0.8
		0.75	0.72
		3.75	0.52
Glycerine	10.14	0.125	0.412
		0.875	0.396
		1.75	0.347
Hydrochloric acid	19.2	0.10	2.56
		0.90	3.04
		3.20	4.5
Nitric acid	19.5	0.10	2.4
		0.90	2.62
		3.90	2.85
Potassium chloride	17.5	0.02	1.57
	18	1.0	1.54
		3.8	1.55
Silver nitrate	12	0.02	1.19
		0.10	1.13
		0.90	1.02
		3.9	0.61
Sodium chloride	15	0.02	1.09
		0.1	1.09
		0.9	1.12
		3.9	1.18
Sulphuric acid	18	0.35	1.53
		2.85	1.85
		4.85	2.20

Table 58

Van der Waals Constants

Substance	a, 10^5 J·m^3/kg-mol^2	b, 10^{-3} m^3/kg-mol	Substance	a, 10^5 J·m^3/kg-mol^2	b, 10^{-3} m^3/kg-mol
Acetone	14.1	98.5	Methane	2.28	27.1
Ammonia	4.22	37.2	Methyl alcohol	15.2	67
Argon	1.36	32.3	Neon	0.22	17.1
Benzene	18.2	115	Nitrogen	1.41	39.2
Ethyl alcohol	12.2	840	Oxygen	1.38	31.8
Ethyl ether	17.5	134	Propane	0.88	84.5
Helium	0.035	23.8	Propyl alcohol	15	101
Hydrogen	2.47	26.6	Water	5.55	30.5
Krypton	2.34	39.9	Xenon	4.15	51
Mercury	8.2	16.7			

Table 59

Heat of Combustion of Selected Fuels

Fuel	W_h, 10^5 J/kg	W_l, 10^5 J/kg
Solid		
Anthracite (grade "A")	320-340	190-270
Brown coal	250-290	100-170
Charcoal (dry)	300	—
Coal, long-flame (Д)	310-320	210-240
Dynamite 75%	—	54
Gunpowder	—	30-31
Peat	220-250	84-110
Shale, combustible	270-330	63-84
Wood	190	100

Table 59 (concluded)

Fuel	W_h, 10^5 J/kg	W_l, 10^5 J/kg
Liquid		
Diesel motor vehicle fuel	—	427
Ethyl alcohol	—	272
Fuel oil	—	390-410
Kerosene, commercial	—	430
Petrol, high grade	—	441
Petrol, third grade	—	436
Gas		
Carbon monoxide	—	130
Coke oven gas	—	160-190
Hydrogen	—	110
Illuminating gas	—	175-210
Natural gas	—	360

Note. The heat of combustion determined without account taken of the losses for the evaporation of the water contained in the fuel is called the *higher heat of combustion* W_h, and with account taken of these losses—the *lower heat of combustion* W_l.

Table 60

Psychrometric Table of Relative Humidity of Air

Reading of dry-bulb thermometer, °C	Difference between readings of dry- and wet-bulb thermometers, °C										
	0	1	2	3	4	5	6	7	8	9	10
0	100	81	63	45	28	11	—	—	—	—	—
2	100	84	68	51	35	20	—	—	—	—	—
4	100	85	70	56	42	28	14	—	—	—	—
6	100	86	73	60	47	35	23	10	—	—	—
8	100	87	75	63	51	40	28	18	7	—	—
10	100	88	76	65	54	44	34	24	14	4	—
12	100	89	78	68	57	48	38	29	20	11	—
14	100	90	79	70	60	51	42	33	25	17	9
16	100	90	81	71	62	54	45	37	30	22	15
18	100	91	82	73	64	56	48	41	34	26	20
20	100	91	83	74	66	59	51	44	37	30	24
22	100	92	83	76	68	61	54	47	40	34	28
24	100	92	84	77	69	62	56	49	43	37	31
26	100	92	85	78	71	64	58	50	45	40	34
28	100	93	85	78	72	65	59	53	48	42	37
30	100	93	86	79	73	67	61	55	50	44	39

Note. The relative humidity is determined by means of a *psychrometer* (wet- and dry-bulb hygrometer) which consists of two thermometers, one with a dry bulb and the other with a bulb covered by a piece of wet material. In order to determine the relative humidity from Table 60 find the intersection of the column corresponding to the measured difference between the readings of the wet- and dry-bulb thermometers, and the line corresponding to the reading of the dry-bulb thermometer.

CHAPTER III

MECHANICAL OSCILLATIONS AND WAVES

FUNDAMENTAL CONCEPTS AND LAWS

1. Harmonic Motion

Motions (or changes of state) which to a certain extent repeat themselves at regular intervals of time are called in physics and engineering *vibrational* or *oscillatory motions* (*vibrations* or *oscillations*).

If the oscillations involve a change only of mechanical quantities (displacement, velocity, density, acceleration, etc.), then we speak of *mechanical oscillations*.

Periodic oscillations are oscillations in which each value of the variable quantity is repeated an endless number of times at regular time intervals. The smallest time interval T which elapses between two successive repetitions of each value of the variable quantity is called the *period of oscillation*.

The quantity $\nu = \frac{1}{T}$ is called the *frequency* of periodic oscillations. The frequency ν is measured in hertz (or cycles per second); 1 hertz (Hz) is the frequency of periodic oscillation whose period is 1 second.

Harmonic motion (*oscillations*) is the periodic variation of a quantity which can be expressed as a sine (or cosine) function:

$$x = A \sin(\omega t + \varphi) \tag{3.1}$$

The positive quantity A in (3.1) is called the *amplitude* of harmonic motion, $(\omega t + \varphi)$—the *phase* (or *phase angle*) of harmonic motion, φ—the *starting phase*, and ω—the *cyclic* (or *angular*) *frequency*:

$$\omega = \frac{2\pi}{T} = 2\pi\nu \tag{3.2}$$

The phase of harmonic motion determines the value of a variable quantity at a given moment of time. The phase is expressed in units of angular measure (radians or degrees). The angular frequency is measured in radians per second (rad/s).

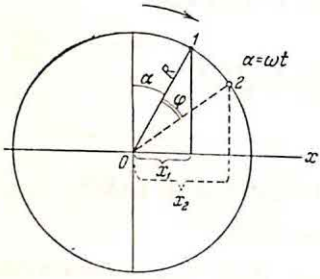

Fig. 24. Harmonic motion of projections of balls describing uniform circular motion

An example of harmonic motion is the motion of the projection of a ball which is in uniform circular motion with an angular velocity ω (Fig. 24). For balls *1* and *2* the displacements of the projections are, respectively,

$$x_1 = R\sin\alpha = R\sin\omega t$$

$$x_2 = R\sin(\alpha + \varphi) = R\sin(\omega t + \varphi)$$

Oscillations with the same frequency but different starting phases are said to be *out of phase* (or to have a phase difference).

The difference between the starting phases is called the *phase difference*. The phase difference between two oscillations of the same frequency is independent of the choice of the zero of time. For example, the phase difference between the projections of balls *1* and *2* (see Fig. 24) is φ for any arbitrary zero of time.

Harmonic motion (oscillations) of a body appears when it is acted upon by a quasi-elastic force. The term *quasi-elastic forces* is used to denote forces that are not elastic in nature, but whose magnitude is proportional to the displacement of the body from the equilibrium position; these forces are always directed towards the equilibrium position. The mathematical expression of a quasi-elastic force is

$$F = -kx \tag{3.3}$$

where k is a coefficient of proportionality called the *quasi-elastic force constant*, and x—the displacement; the minus sign denotes that the force is directed towards the equilibrium position.

Any kind of periodic motion can be represented to any degree of accuracy by a sum of harmonic motions.*

2. The Pendulum

A *physical pendulum* is a rigid body which is suspended from some point above its centre of gravity. A body thus suspended can perform oscillations. The pendulum is called a *simple* (or *mathematical*) *pendulum* if the entire mass of the body can be regarded as concentrated at one point. A sufficiently close approximation of a simple pendulum is a small ball (called a pendulum bob) suspended from an inextensible string, if the friction of the air and the pendulum support are negligible, and the dimensions of the ball are small compared with the length of the string. For small angular displacements the oscillations of a simple pendulum may be considered harmonic. All the formulas given below relate exactly to such oscillations.

The period of the simple pendulum is given by the formula:

$$T=2\pi\sqrt{\frac{l}{g}} \tag{3.4}$$

where l is the length of the pendulum, and g—the acceleration due to gravity.

The oscillations of a bob suspended from a spring can be considered harmonic if the amplitude of oscillations is within the limits of validity of Hooke's law (see p. 53), and frictional forces are negligible.

The period of the bob is

$$T=2\pi\sqrt{\frac{m}{k}} \tag{3.5}$$

where m is the mass of the bob, and k—the *coefficient of elasticity of the spring*, equal numerically to the force required to stretch the spring by 1 cm.**

---

* It is shown in mathematical analysis that any periodic motion can be represented by an infinite sum of harmonic motions, called a *harmonic series*.

** Formula (3.5) applies not only to the case of a bob suspended from a spring, but also to all cases for which the relation (3.3) is valid.

A *torsion pendulum* is a body which performs rotary-oscillatory motion under the action of a spring (for example, the balance wheel in watches and clocks). Under certain conditions (when the amplitude of oscillation is sufficiently small and frictional forces are negligible) such motion can also be considered harmonic.

The period of a torsion pendulum is

$$T = 2\pi\sqrt{\frac{J}{D}} \tag{3.6}$$

where J is the moment of inertia of the body about the axis of rotation, and D—the *torsional rigidity*, equal numerically to the torque required to turn the body through unit angle.

The period of a physical pendulum is

$$T = 2\pi\sqrt{\frac{J}{mga}} \tag{3.7}$$

where J is the moment of inertia of the body about an axis passing through the point of support, a—the distance from the centre of gravity to this axis, m—the mass of the body, and g—the acceleration due to gravity.

The quantity

$$l = \frac{J}{ma}$$

is called the *reduced length of a physical pendulum*, which is equal to the length of a mathematical pendulum whose period of oscillations coincides with that of the given physical pendulum.

3. Free and Forced Oscillations

The oscillations which a body performs when it is in some way displaced from equilibrium and then released are called *free* (or *natural*) *oscillations*.

If the free oscillations of a body are caused only by a quasi-elastic force, then they will be harmonic.

The oscillations of a body due to the simultaneous action of a quasi-elastic force and a frictional force (which is proportional to the instantaneous velocity, $F_{fr} = -rv$*,

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* The minus sign denotes that the vectors of velocity and force are oppositely directed.

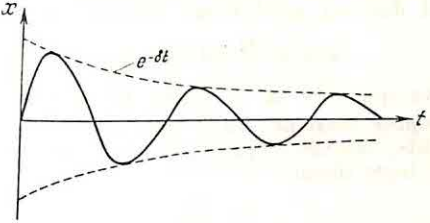

Fig. 25. Damped oscillations

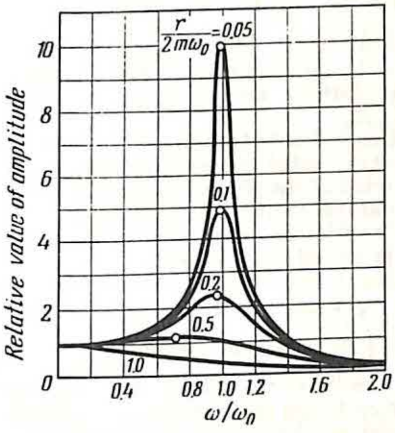

Fig. 26. Resonance curves for various degrees of damping.

The axis of ordinates shows the relative values of the displacement amplitude $\frac{B}{F_0/k}$, where B is the displacement amplitude, and $\frac{F_0}{k}$ — the static displacement equal to the displacement which would be induced by a constant force having a value equal to the amplitude of the acting force. The axis of abscissas shows the relative changes in the frequency $\frac{\omega}{\omega_0}$, where $\omega_0 = \sqrt{\frac{k}{m}}$ is the frequency of the free oscillations in the absence of friction. The curves relate to different values of $\frac{r}{2m\omega_0}$. The dots show the position of the maximum value of the displacement amplitude where v is the velocity) are called *damped oscillations*. The equation of damped oscillations has the form

$$x = Ae^{-\delta t} \sin(\omega t + \varphi) \tag{3.8}$$

The positive quantity A is called the *initial amplitude*, δ—the *damping constant*, $Ae^{-\delta t}$—the *instantaneous value of the amplitude*, ω—the *angular frequency*, and e—the base of natural logarithms;

$$\delta = \frac{r}{2m} \tag{3.9}$$

and

$$\omega = \sqrt{\frac{k}{m} - \frac{r^2}{4m^2}} \tag{3.10}$$

where r is the *frictional force constant*, m—the mass of the body, and k—the quasi-elastic force constant.

Damped oscillations are depicted graphically by the curve in Fig. 25.

The oscillations of a body under the action of a periodic driving force are called *forced oscillations*.

When the period of the sinusoidal driving force approaches the period of natural oscillations of the body, the amplitude of the forced oscillations increases sharply (Fig. 26). This phenomenon is called *resonance*.

If the frictional forces are large (strong damping), then the resonance is weak (see Fig. 26), or does not manifest itself at all $\left(\text{for example, when } \frac{r}{2m\omega_0} > 1\right)$.

Undamped oscillations which are maintained by the action of driving forces on a system at definite moments of time determined by the system itself are called *self-induced*, or *self-maintained oscillations*.

In self-maintained oscillations the system itself controls the driving action (for example, the pendulum in a clock).

4. Summation of Harmonic Oscillations

When a body simultaneously performs two (or more) oscillations, its resulting displacement at any moment of time equals the vector sum of all the individual displacements.

Upon the summation of two harmonic oscillations having the same frequencies and directions:

$$x_1 = A_1 \sin(\omega t + \varphi_1)$$
$$x_2 = A_2 \sin(\omega t + \varphi_2)$$

the amplitude of the resultant displacement A_r is found according to the rule of the parallelogram as shown in Fig. 27. The resultant displacement in this case will be

$$x_r = A_r \sin(\omega t + \varphi_r)$$

where

$$A_r = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\varphi_2 - \varphi_1)}$$

$$\tan \varphi_r = \frac{A_1 \sin \varphi_1 + A_2 \sin \varphi_2}{A_1 \cos \varphi_1 + A_2 \cos \varphi_2}$$

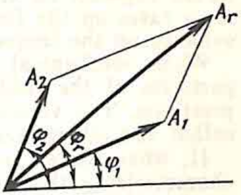

Fig. 27. Summation of displacement amplitudes of harmonic oscillations having the same direction

When a body simultaneously performs two harmonic oscillations (with the same frequency) in mutually perpendicular directions, its displacements are determined by the equations

$$x = A_1 \sin \omega t$$
$$y = A_2 \sin(\omega t + \varphi)$$

and the trajectory of its motion is described by the equation of an ellipse (Fig. 28):

$$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy}{A_1A_2} \cos \varphi = \sin^2 \varphi$$

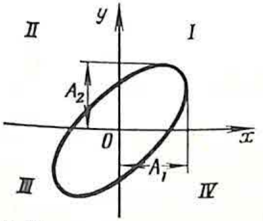

Fig. 28. Summation of mutually perpendicular harmonic oscillations

When $A_1 = A_2$ and $\varphi = 90°$, the body travels along a circle having the radius A_1. When $\varphi = 0$, the body travels along a straight line passing through quarters *I* and *III*; when $\varphi = \pi$, the body travels along a straight line passing through quarters *II* and *IV*.

5. Waves

The process of propagation of any kind of disturbances (i.e. changes of state) is called *waves*. For example, if we strike one end of a metal bar, a local compression arises at that end and passes along the bar with a definite velocity.

The velocity with which the disturbance advances in space is called the *wave velocity*. The velocity of mechanical waves depends on the properties of the medium, and in some cases on the frequency. The dependence of the wave velocity on the frequency is called *dispersion of the velocity*.

When mechanical waves propagate in a medium, the particles of the medium vibrate about their equilibrium positions. The velocity of the particles of the medium is called the *vibrational velocity*.

If, when waves propagate in a medium, the parameters characterizing the medium (for example, density, particle displacement, pressure, etc.) vary at any arbitrary point in space according to a sine function. the waves are called *sine waves*.

An important characteristic of sine waves is the wavelength. The *wavelength* (λ) is the distance travelled by a wave in one period:

$$\lambda = vT \tag{3.11}$$

$$\nu = \frac{v}{\lambda} \tag{3.12}$$

where v is the wave velocity, ν—the frequency, and T—the period.

The mathematical expression

$$x = A \sin \omega \left(t - \frac{r}{v} \right) = A \sin (\omega t - kr)$$

which describes the variation of a parameter of the medium through which sine waves are propagating is called the *equation of plane harmonic waves*.* In this expression A is the *amplitude of the wave*, ω—the cyclic frequency, r—the distance from the source initiating the wave to the point of interest in the medium, v—the velocity of the wave, $k =$

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* Here x denotes any parameter characterizing the state of the medium (for example, pressure, temperature, etc.).

$= \frac{2\pi}{\lambda}$—the *wave number*, and the expression $\omega t - kr$ is called the *phase of the wave*.

A surface all of whose points are in the same phase is called a *wave front*, or a *wave surface*.

According to the shape of the wave surface we distinguish between *plane* (plane wave surfaces), *cylindrical* (cylindrical wave surfaces), and *spherical* (spherical wave surfaces) waves.

The equation of cylindrical waves is

$$x = \frac{A}{\sqrt{r}} \sin(\omega t - kr)$$

and of spherical waves

$$x = \frac{A}{r} \sin(\omega t - kr)$$

If the particles of a medium are displaced parallel to the direction of propagation of a wave, the latter is called *longitudinal*; if the particles are displaced in a plane perpendicular to the direction of propagation, the wave is called *transverse*. Mechanical waves in liquid and gaseous media are longitudinal, whereas in solids both longitudinal and transverse waves are possible.

The velocity of longitudinal waves in a rod is given by the formula

$$v_{\mathrm{l,\,r}} = \sqrt{\frac{E}{\rho}} \qquad (3.13)$$

where E is Young's modulus, and ρ—the density.

The velocity of longitudinal waves in a solid whose transverse dimensions are much greater than the wavelength is

$$v_{\mathrm{l,\,s}}^{\infty} = \sqrt{\frac{E}{\rho}\,\frac{1-\mu}{(1+\mu)(1-2\mu)}} \qquad (3.14)$$

where ρ is the density of the substance, E—Young's modulus, and μ—Poisson's ratio (see Table 17).

The velocity of longitudinal waves in thin plates is

$$v_{\mathrm{l,\,pl}} = \sqrt{\frac{E}{\rho(1-\mu^2)}} \qquad (3.15)$$

The velocity of longitudinal waves in liquids is given by the formula

$$v_{\mathrm{l,\,lq}}=\sqrt{\frac{\gamma}{\rho\beta_{\mathrm{is}}}} \qquad (3.16)$$

where β_{is} is the *isothermal compressibility**, and $\gamma=\frac{c_p}{c_v}$.

The velocity of transverse (shear) waves is given by

$$v_{\mathrm{t}}=\sqrt{\frac{G}{\rho}} \qquad (3.17)$$

where G is the shear modulus (see p. 55).

The velocity of sound waves in gases is expressed by the formula:

$$v_{\mathrm{s,\,g}}=\sqrt{\gamma\frac{p}{\rho}} \qquad (3.18)$$

where $\gamma=\frac{c_p}{c_v}$, and p is the pressure.

Formula (3.18) may be applied to ideal gases; in this case it can be written in the form

$$v_{\mathrm{s,\,g}}=\sqrt{\gamma\frac{RT}{\mu'}} \qquad (3.19)$$

where μ' is the molecular weight.

Waves on the surface of a liquid are neither transverse nor longitudinal. The particles of water describe more complex motion in surface waves (Fig. 29).

The velocity of surface waves** is given by

$$v_{\mathrm{sur}}=\sqrt{\frac{g\lambda}{2\pi}+\frac{2\pi\alpha}{\lambda\rho}} \qquad (3.20)$$

where g is the acceleration due to gravity, λ—the wavelength, α—the surface tension, and ρ—the density.

Formula (3.20) applies when the depth of the liquid is not less than 0.5λ.

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* For compressibility see p. 55. In isothermal compressibility compression takes place at constant temperature.

** Formula (3.20) applies to waves on a liquid-gas interface when the density of the liquid is much greater than that of the gas.

When the depth of the liquid h is small (less than 0.5λ), the velocity is expressed by the formula:

$$v_{\text{sur}} = \sqrt{gh} \tag{3.21}$$

Wave motion is accompanied by the transfer of energy; the particles of the medium, however, are not carried along the direction of propagation of the wave, but only oscillate about their equilibrium positions (if the waves are of small amplitude and the medium is non-viscous).

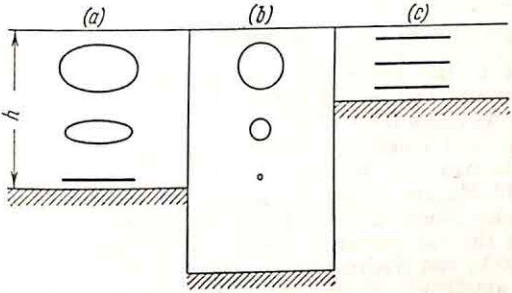


Fig. 29. Trajectories of particles of water in propagation of surface waves:
(a) in shallow water; (b) in deep water $\left(\text{the ratio } \frac{2\pi h}{\lambda} \text{ is very large}\right)$; (c) in very shallow water $\left(\text{the ratio } \frac{2\pi h}{\lambda} \text{ is very small}\right)$

The mean quantity of energy transmitted per second across 1 square centimetre of wave surface is called the *intensity of the wave*.

Intensity is expressed in W/cm^2 or erg/cm^2·s.

The intensity of sound waves is sometimes called *sound intensity*.

When mechanical waves travel through a medium, the velocity and acceleration of the medium particles vary according to the same harmonic law as the displacement does.

If the amplitude of displacement of the particles is x_0 when a plane wave of cyclic frequency ω passes through a medium, the amplitude of the vibrational velocity will be

$$u_0 = \omega x_0 \tag{3.22}$$

the amplitude of acceleration

$$a_0 = \omega^2 x_0 \tag{3.23}$$

and the intensity

$$I = \frac{1}{2}\rho v u_0^2 \tag{3.24}$$

where ρ is the density of the medium, and v—the wave velocity.

6. Sound

Sound is the name given to mechanical vibrations in elastic media and bodies (solid, liquid, and gaseous) whose frequencies range from 17-20 to 20 000 Hz. These frequencies of mechanical vibrations can produce the sensation of sound in the human ear. Mechanical vibrations with frequencies below 17 Hz are called *infrasonic* (or *subsonic*), and with frequencies above 20 000 Hz are called *ultrasonic*.

When the ear perceives sound, it distinguishes its loudness, pitch, and timbre. The *loudness* of sound is determined by the amplitude of its vibrations, the *pitch* by their frequency, and the *timbre* by the amplitude of the vibrations of the overtones (vibrations with higher frequencies).

The change in pressure in a medium upon the propagation of sound waves in it in comparison with the pressure in the absence of these waves is called the *sound pressure*. The following relation exists between the amplitude of sound pressure Δp_0 and the maximum vibrational velocity:

$$\Delta p_0 = \rho v u_0 \tag{3.25}$$

The intensity of plane sound waves diminishes owing to absorption in a medium according to the law

$$I_x = I_0 e^{-2\alpha x} \tag{3.26}$$

where I_0 is the intensity of the waves entering the medium, and I_x—their intensity after travelling the path x.

The quantity α determining the degree of attenuation of sound waves is called the *sound absorption coefficient* (with respect to amplitude).

The intensity of sound corresponds to the subjective sensation of loudness. Below a certain minimum intensity, called the *threshold of audibility*, sound is no longer audible to the human ear. The threshold of audibility is different for sounds of different frequencies. Sound of great intensity produces a painful sensation in the ear. The smallest intensity of sound causing such a sensation is called the *threshold of discomfort.*

A change in intensity (intensity level) is expressed in decibels (dB).

The number of decibels equals the common logarithm of the ratio of the intensities multiplied by 10, i.e. $10 \log \left(\frac{I}{I_0}\right)$.

As a rule, in acoustics I_0 is taken equal to 10^{-9} erg/cm²·s, which is approximately equal to the intensity at the threshold of audibility at 1000 Hz.

TABLES AND GRAPHS

Table 61

Sound Intensity and Sound Pressure

Decibels	I, W/m²	Δp, N/m²	Examples of sound of the indicated intensity
0	10^{-12}	0.00002	Threshold of audibility of the human ear
10	10^{-11}	0.000065	Rustle of leaves. Low whisper at a distance of 1 m
20	10^{-10}	0.0002	Quiet garden
30	10^{-9}	0.00065	Quiet room. Average sound level in an auditorium. Violin playing pianissimo
40	10^{-8}	0.002	Low music. Noise in a living room
50	10^{-7}	0.0065	Loudspeaker at low volume. Noise in a restaurant or office with open windows
60	10^{-6}	0.02	Radio turned on loud. Noise in a store. Average level of speech at a distance of 1 m
70	10^{-5}	0.0645	Noise of a truck motor. Noise inside a tramcar
80	10^{-4}	0.20	Noisy street. Typists' room
90	10^{-3}	0.645	Automobile horn. Large symphony orchestra playing fortissimo
100	10^{-2}	2.0	Riveting machine. Automobile siren
110	10^{-1}	6.45	Pneumatic hammer
120	1	20	Jet engine at a distance of 5 m. Loud thunderclaps
130	10	64.5	Threshold of pain, sound is no longer audible

Table 62

Velocity of Sound in Pure Liquids and Oils

Liquid	t, °C	v, m/s	α, m/s·K
Pure liquids			
Acetone	20	1192	−5.5
Aniline	20	1656	−4.6
Benzene	20	1326	−5.2
Ethyl alcohol	20	1180	−3.6
Glycerine	20	1923	−1.8
Kerosene	34	1295	—
Mercury	20	1451	−0.46
Methyl alcohol	20	1123	−3.3
Ordinary water	25	1497	2.5
Sea water	17	1510-1550	—
Oils			
Cedar	29	1406	—
Eucalyptus	29.5	1276	—
Gasoline	34	1250	—
Linseed	31.5	1772	—
Olive	32.5	1381	—
Peanut	31.5	1562	—
Rapeseed	30.8	1450	—
Spindle	32	1342	—
Transformer	32.5	1425	—

Note. The velocity of sound in liquids decreases with a rise in temperature (with the exception of water). The velocity at temperatures other than those given in the table can be computed from the formula: $v_t = v + \alpha\,(t - t_0)$, where v is the velocity given in the table, α—the temperature coefficient given in the last column of the table for pure liquids, t—the temperature for which the velocity is calculated, and t_0—the temperature indicated in the table.

Table 63

Velocity of Sound in Solids
(at 20 °C)

Material	$v_{l,r}$, m/s	v_l^{∞}, m/s	v_t, m/s
Aluminium	5080	6260	3080
Brass	3490	4430	2123
Caoutchouc	—	1479	—
Copper	3710	4700	2260
Cork	500	—	—
Ebonite	1570	2405	—
Glass, crown	5300	5660	3420
Ditto, heavy crown	4710	5260	2960
Ditto, heavy flint	3490	3760	2220
Ditto, light flint	4550	4800	2950
Ditto, quartz	5370	5570	3515
Ice	3280	3980	1990
Iron	5170	5850	3230
Lead	2640	3600	1590
Limestone	—	6130	3200
Marble	—	6150	3260
Mica	—	7760	2160
Nickel	4785	5630	2960
Plaster of Paris	—	4970	2370
Plexiglas	—	2670	1121
Polystyrene	—	2350	1120
Porcelain	4884	5340	3120
Rubber	46	1040	27
Sandstone	—	3700–4900	—
Slate	—	5870	2800
Steel, carbon	5050	6100	3300
Tin	2730	3320	1670
Zinc	3810	4170	2410

Note. $v_{l,r}$—velocity of longitudinal waves in rods
v_l^{∞}—velocity of longitudinal waves in infinite medium
v_t—velocity of transverse (shear) waves in infinite medium.

Table 64

Properties of Earth at Various Depths H and Velocity of Seismic Waves

H, km	ρ, 10^3 kg/m^3	v_l, 10^3 m/s	v_t, 10^3 m/s	p, 10^{11} N/m^2	g, m/s^2
33	3.32	8.18	4.63	0.009	9.85
100	3.38	8.18	4.63	0.031	9.89
200	3.47	8.29	4.63	0.065	9.92
500	3.89	9.65	5.31	0.174	9.99
1000	4.68	11.42	6.36	0.392	9.95
2000	5.24	12.79	6.93	0.88	9.86
4000	10.8	9.51	—	2.40	8.00
5000	11.5	10.44	—	3.18	6.13

Note. Mechanical waves propagating in the Earth's crust are called *seismic waves.*

Seismic waves can be longitudinal v_l (compressional) and transverse v_t (shear); the density ρ, pressure p, and acceleration of gravity g are also indicated.

Table 65

Velocity of Sound in Gases (at pressure of 1 atm)

Gas	t, °C	v, m/s	α, m/s·K
Air	0	331	0.59
Ammonia	0	415	—
Benzene (vapour)	97	202	0.3
Carbon dioxide	0	259	0.4
Ethyl alcohol	97	269	0.4
Helium	0	965	0.8
Hydrogen	0	1284	2.2
Methyl alcohol	97	335	0.46
Neon	0	435	0.8
Nitrogen	0	334	0.6
Oxygen	0	316	0.56
Water vapour	134	494	—

Notes. 1. The velocity of sound in gases at constant pressure increases with the temperature. The temperature coefficient of the velocity α is therefore given in the table, so that the velocity can be computed for other temperatures (see note to Table 62).

2. At high frequencies (or low pressures) the velocity of sound depends on the frequency. The values given in the table are for frequencies and pressures at which the velocity is practically independent of the frequency.

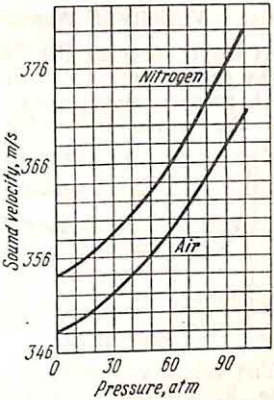


Fig. 30. Pressure dependence of sound velocity in air and nitrogen. The curves relate to 20 °C and the frequency range from 200 to 500 kHz

Table 66

Scale of Mechanical Waves

Frequency, Hz	Name	Emitter	Application
0.5-20	Infrasound	Vibrations of water in large reservoirs, heartbeats	Weather forecasting, diagnostics of heart ailments
$20-2\times10^4$	Audible sound	Voices of human beings and animals, musical instruments, whistles, sirens, loudspeakers, etc.	Communications and signalling, measurement of distances (sound ranging)
$2\times10^4-10^{10}$	Ultrasound	Magnetostrictive and piezoelectric oscillators, Galton whistle; also emitted by some animals and insects (bats, crickets, locusts, etc.)	Hydrolocation, cleaning of parts, flaw detection in parts and structural members, acceleration of chemical reactions, medical and biological research, molecular physics
10^{11} and above	Hypersound	Thermal vibrations of molecules	Scientific research

Velocity of Waves on the Surface of Water

At small wavelengths (less than 2 cm) the decisive factor is the surface tension; such waves are called *capillary waves*.

At greater wavelengths the decisive factor is gravity, and the waves are called *heavy*, or *gravity waves*.

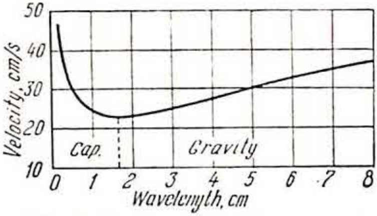


Fig. 31. Dispersion of surface waves ($h > 0.5\lambda$)

The velocity of surface waves depends on the wavelength [Fig. 31 and Eq. (3.20)] if the depth of the liquid is sufficiently great ($h > 0.5\lambda$).

Loudness of Audibly Perceived Sounds

Figure 32 gives curves of the intensity of sounds of equal loudness. The upper curve corresponds to the threshold of

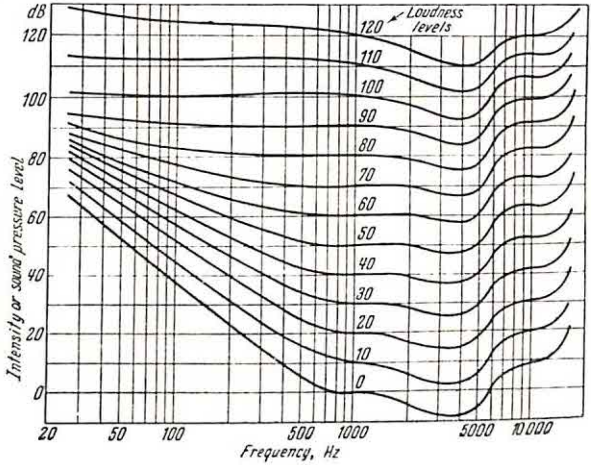


Fig. 32. Loudness levels

pain, the lower to the threshold of audibility. The frequencies are plotted using a logarithmic scale.

Displacement and Acceleration of Water Particles upon Passage of Sound Waves of Various Intensities

Figures 33 and 34 give the amplitudes of the displacement and acceleration computed by formulas (3.22) and (3.23).

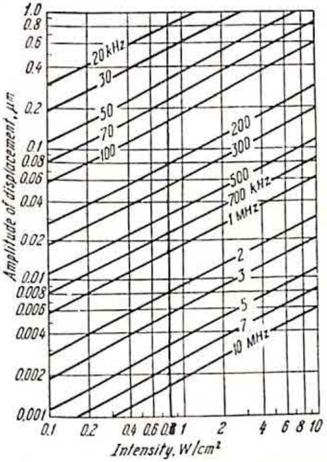


Fig. 33. Displacement of water particles in propagation of sound waves

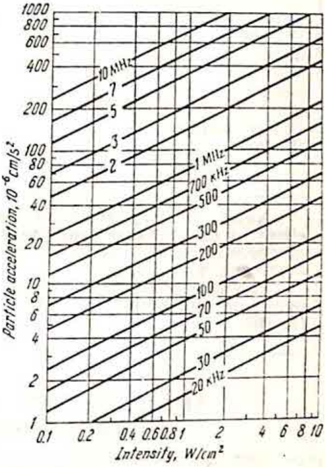


Fig. 34. Acceleration of water particles in propagation of sound waves

The calculations have been performed for $\rho v = 1.5 \times 10^5$ g/cm²·s. Logarithmic scales are used on both axes.

Table 67

Reflection Coefficient of Sound Waves for Various Interfaces (at normal incidence, %)

Material	Aluminium	Copper	Glass	Mercury	Nickel	Steel	Transformer oil	Water
Aluminium	0	18	2	1	24	21	74	72
Copper		0	19	13	0.8	0.3	88	87
Glass			0	4	34	31	67	65
Mercury				0	19	16	76	75
Nickel					0	0.2	90	89
Steel						0	89	88
Transformer oil							0	0.6
Water								0

Notes. 1. The reflection coefficient is the ratio of the intensities of the reflected and incident sound waves. The reflection coefficient is the same for sound passing from mercury into steel and vice versa.

2. Upon reflection from a plate the reflection coefficient depends on the ratio of the thickness of the plate to the wavelength.

Table 68

Absorption Coefficient of Sound in Air (α, 10^{-4} cm^{-1} at 20 °C)

Frequency, kHz	Relative humidity of air, %				
	10	20	40	60	80
1	0.13	0.06	0.03	0.03	0.03
2	0.47	0.23	0.10	0.09	0.08
4	1.27	0.82	0.38	0.24	0.20
6	1.87	1.61	0.84	0.54	0.39
8	2.26	2.48	1.45	0.96	0.69
10	2.53	3.28	2.20	1.47	1.08

Note. The values in the table relate to pressures close to standard.

Table 69

Sound Absorption of Selected Materials

Material	Frequency, Hz					
	125	250	500	1000	2000	4000
Brick wall	0.024	0.025	0.032	0.041	0.049	0.07
Cotton material	0.03	0.04	0.11	0.17	0.24	0.35
Glass, sheet	0.03	—	0.027	—	0.02	—
Glass wool (9 cm thick)	0.32	0.40	0.51	0.60	0.65	0.60
Hair felt (25 mm thick)	0.18	0.36	0.71	0.79	0.82	0.85
Marble	0.01	—	0.01	—	0.015	—
Plaster, gypsum	0.013	0.015	0.020	0.028	0.04	0.05
Ditto, lime	0.025	0.045	0.06	0.085	0.043	0.058
Rug with nap	0.09	0.08	0.21	0.27	0.27	0.37
Wooden planking	0.10	0.11	0.11	0.08	0.082	0.11

Note. The *absorption coefficient* of sound (upon reflection) is the ratio of the energy absorbed to the energy incident on the reflecting surface.

Table 70

Absorption of Sound in Selected Liquids

Liquid	t, °C	Range of frequencies f, MHz	$\frac{\alpha}{f^2}$, 10^{-17} s^2/cm
Acetone	25	4-20	50
Benzene	20	1-200	850-900
Castor oil	18.5	3	11 000
Ethyl ether	25	10	140
Glycerine	26	4-20	1700
Kerosene	25	6-20	110
Mercury	20	0.5-1000	5.5
Nitrogen	−199	44.5	11
Petroleum	25	10	~ 100
Turpentine	25	10	150
Water	20	1-200	25

Note. The values in the table relate to a pressure of from 1 to 20 atm. At these pressures the absorption does not practically depend on the pressure.

Table 71

Absorption of Sound Waves in Sea Water (at 15-20 °C)

f, kHz	20	24	100	200	230	480	940
α, 10^{-4} cm^{-1}	0.023	0.050	0.37	0.69	1.25	2.00	2.90

CHAPTER IV

ELECTRICITY

A. THE ELECTROSTATIC FIELD

FUNDAMENTAL CONCEPTS AND LAWS

Two kinds of electric charge exist—positive and negative. Charges generated on glass rubbed with silk are called *positive*, and charges generated on ebonite rubbed with fur are called *negative*.

Like charges repel each other, and unlike charges attract each other.

Electrons are the carriers of negative charges in an atom, and protons forming part of an atomic nucleus (see p. 227) are carriers of positive charges. The sum of the positive and negative charges in an atom equals zero; the charges are distributed so that the atom as a whole is neutral.

The process of *electrization* consists in the non-uniform distribution of the negative and positive charges between bodies (for example, upon electrization by friction or in a galvanic cell, see p. 142) or between different parts of the same body (for example, in electromagnetic induction, see p. 168).

Electric charges are not created and do not vanish; they can travel from one body to another or move inside a body or molecules of atoms (the *law of conservation of electric charges*).

Charges in various media can be carried by electrons detached from their atoms (for example, in metals), by ions—parts of molecules or atoms having positive or negative charges (for example, in electrolytes and gases), and molions—colloidal particles in a liquid having charges.

The magnitude of any charge is a multiple (in absolute value) of the charge of an electron. The charge of the latter has the smallest value; this smallest portion of charge is called an *elementary charge*. The charge of a proton equals that of an electron in absolute value.

Interaction of charges. Electric field. The law of interaction of point charges (*Coulomb's law*) is written as follows:

in the CGSE system

$$F = \frac{q_1 q_2}{\varepsilon_a r^2} \tag{4.1}$$

in the SI system

$$F = \frac{q_1 q_2}{4\pi \varepsilon_a r^2} \tag{4.2}$$

where F is the force of interaction, q_1 and q_2—the magnitudes of the charges, ε_a—the absolute permittivity of the medium, and r—the distance between the point charges.

The numerical value of ε_a can be expressed in relative units (relative to the absolute value of the permittivity of a vacuum ε_0).

The quantity $\varepsilon = \frac{\varepsilon_a}{\varepsilon_0}$ is called the *relative permittivity*; it shows how many times the interaction between charges in an infinite homogeneous medium is less than in a vacuum; $\varepsilon = \frac{\varepsilon_a}{\varepsilon_0}$ is often called simply the *permittivity* (or *dielectric constant*).

The numerical value of the quantity ε_0 and its dimension depend on what system of units is used; the value of ε does not depend on the system of units.

In the CGSE system $\varepsilon_0 = 1$ (this quantity is the fourth fundamental unit); in the SI system

$$\varepsilon_0 = \frac{1}{4\pi \times 9 \times 10^9} \frac{\text{farad}}{\text{metre}} = 8.85 \times 10^{-12} \text{ F/m} \tag{4.3}$$

(in this system ε_0 is a derived quantity).

In the CGSE system the unit of charge is the charge which acts in a vacuum on an equal charge at a distance of 1 cm from it with a force of 1 dyne. In the SI system the unit of charge is the coulomb (C):

$$1\ \mathrm{C} = 2.997\,93 \times 10^9\ \text{CGSE units} \cong 3 \times 10^9\ \text{CGSE units}$$

The elementary charge $e = 4.8 \times 10^{-10}$ CGSE units.

If the action of forces on stationary electric charges is detected in a space, an *electric field* is said to exist in it.

Electrically charged bodies are always surrounded by an electric field. The field of fixed charges is called *electrostatic*. The *intensity* of an electric field at a given point is numerically equal to the force acting on a unit positive charge placed at this point:

$$E = \frac{F}{q} \tag{4.4}$$

Intensity is a vector quantity. The direction of the intensity vector coincides with that of the force acting on the positive charge. The intensities of the fields of two and more different electric charges are added according to the rule of the parallelogram, i.e. vectorially (see Introduction).

All the following formulas are given in the CGSE and SI systems.

The intensity of the electric field of a point charge is

$$E = \frac{q}{\varepsilon r^2}, \qquad E = \frac{q}{4\pi\varepsilon_0\varepsilon r^2} \tag{4.5}$$

where r is the distance from the point at which the intensity is being determined to the point where the charge q is.

The intensity of the electric field of a uniformly charged plane is

$$E = \frac{2\pi\sigma}{\varepsilon}, \qquad E = \frac{\sigma}{2\varepsilon_0\varepsilon} \tag{4.6}$$

where σ is the charge per unit of surface area.

The intensity of the electric field of a uniformly charged sphere is

$$E = \frac{q}{\varepsilon r^2}, \qquad E = \frac{q}{4\pi\varepsilon_0\varepsilon r^2} \tag{4.7}$$

where r is the distance from the point at which the intensity is being determined to the centre of the sphere.

The intensity of the electric field of a charged cylinder is

$$E = \frac{q'}{\varepsilon r}, \qquad E = \frac{q'}{2\pi\varepsilon_0\varepsilon r} \tag{4.8}$$

where q' is the charge per unit length of the cylinder, and r—the distance from the point at which the intensity is being determined to the axis of the cylinder.

The vector quantity

$$D = \varepsilon_a E$$

is called the *induction* of an electric field.

A line, a tangent to which at each point coincides in direction with the intensity vector, is called a *line of force* of an electric field. The arrangement of the lines of force in electric fields having various structures is shown in Figs. 35-37.

Work and voltage. When a charge is displaced under the influence of an electric field, *work* is performed. The work in an electrostatic field does not depend on the shape of the path along which the charge moves.

A charge placed at any point of an electric field has potential energy.

The scalar quantity numerically equal to the potential energy of a unit positive charge placed at a point is called the *potential* at this point. The magnitude of the potential depends on the selection of the point having a zero potential; the point with a zero potential can be chosen arbitrarily. It is usually assumed in physics that the potential equals zero at an infinitely remote point. In electrical engineering it is assumed that the surface of the Earth has a potential equal to zero.

The potential difference between two points of an electric field is called the *voltage* (U). The voltage numerically equals the work done by electrical forces when a unit positive charge is moved between two points.

The work of displacement of a charge in an electrostatic field is

$$A = qU \tag{4.9}$$

Voltage in the SI system is measured in volts (V). 1 volt is the potential difference between two points when work equal to 1 joule is needed to move 1 coulomb of positive charge between them.

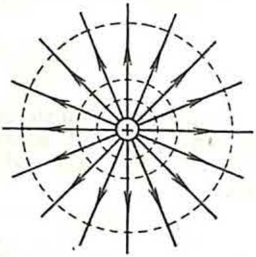
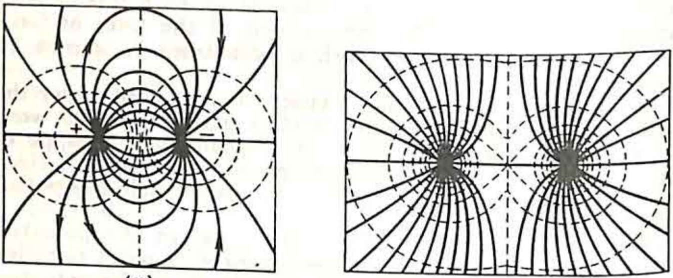


Fig. 35. Lines of force of a point electric charge



(a)

(b)

Fig. 36. Lines of force of:
a) field of two unlike point charges; (b) field of two like point charges

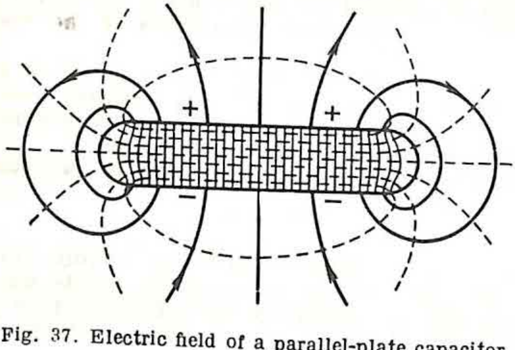


Fig. 37. Electric field of a parallel-plate capacitor

A surface all of whose points have an identical potential is called an *equipotential surface*. In Figs. 35-37 the equipotential surfaces are shown by dash lines.

The lines of force of an electrostatic field are perpendicular to the equipotential surfaces. No work is done by the electric forces in moving a charge from one point to another along an equipotential surface. Let A and B be two points of the field; then the following approximate relation exists between the intensity of the field at the point A and the potential difference between these points:*

$$E_A = -\frac{\Delta U}{\Delta l} \tag{4.10}$$

where ΔU is the potential difference between the close-lying points A and B, and Δl—the distance along the line of force between the equipotential surfaces passing through these points.

The quantity $-\dfrac{\Delta U}{\Delta l}$ is called the *potential gradient*.

If the electric field is homogeneous, i.e. if the intensity is constant in magnitude and direction at all points of the field (for example, in a parallel-plate capacitor), then

$$E = -\frac{U}{l} \tag{4.11}$$

where l is the length of the line of force.

In the SI system the intensity is expressed in volt/metre (V/m). 1 V/m is equal to the intensity of a homogeneous field in which the potential difference between the ends of a line of force 1 m long is equal to 1 V.

Capacitance. Two conductors with an electric field between them whose lines of force emanate from one conductor and terminate on the other form a *capacitor*; the conductors themselves are called the *capacitor plates*.

In a simple capacitor the two plates carry opposite charges of equal magnitude.

The *capacitance* of a capacitor is the ratio of the charge on one of the plates to the potential difference between the plates, i.e.

$$C = \frac{q}{U} \tag{4.12}$$

---

* The exact formula is $E = -\lim\limits_{\Delta l \to 0} \frac{\Delta U}{\Delta l} - \frac{dU}{dl}$.

The SI unit of capacitance is the farad (F). 1 farad is equal to the capacitance of a capacitor the potential difference between whose plates is equal to 1 V when the charge (on one of the plates) is 1 C.

According to the shape of the conducting surfaces capacitors are called *parallel-plate*, *cylindrical*, and *spherical*.

The capacitance of a parallel-plate capacitor is

$$C=\frac{\varepsilon S}{4\pi d}, \qquad C=\frac{\varepsilon_0\varepsilon S}{d} \tag{4.13}$$

where S is the surface area of one plate (the smaller one if they are unequal), d—the distance between the plates, and ε—the dielectric constant of the material between the plates.

The capacitance of a cylindrical capacitor and of a coaxial cable is

$$C=\frac{\varepsilon l}{2\ln\frac{b}{a}}, \qquad C=\frac{2\pi\varepsilon_0\varepsilon l}{\ln\frac{b}{a}} \tag{4.14}$$

where b is the radius of the outer cylinder, a—the radius of the inner cylinder, and l—the length of the capacitor.

The capacitance of a spherical capacitor is

$$C=\frac{\varepsilon}{\frac{1}{a}-\frac{1}{b}}, \qquad C=\frac{4\pi\varepsilon_0\varepsilon}{\frac{1}{a}-\frac{1}{b}} \tag{4.15}$$

where a and b are the radii of the inner and outer spheres.

The capacitance of a two-wire line is

$$C=\frac{\varepsilon l}{4\ln\frac{d}{a}}, \qquad C=\frac{\pi\varepsilon_0\varepsilon l}{\ln\frac{d}{a}} \tag{4.16}$$

where d is the distance between the axes of the parallel wires, a—their radius, and l—their length.

If capacitors of separate capacitances $C_1, C_2, C_3, \ldots, C_n$ are connected in parallel, the capacitance of the whole system is

$$C_{\text{par}}=C_1+C_2+C_3+\ldots+C_n \tag{4.17}$$

for a system of capacitors connected in series the capacitance is

$$\frac{1}{C_{\text{ser}}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots+\frac{1}{C_n} \tag{4.18}$$

The energy stored in a charged capacitor is given by the formula

$$W=\frac{1}{2}CU^2 \qquad (4.19)$$

The space in which an electric field exists contains stored energy. The energy in unit volume of a homogeneous field (energy density) can be computed by the formula

$$w_e=\frac{\varepsilon E^2}{8\pi}, \qquad w_e=\frac{1}{2}\varepsilon_0\varepsilon E^2 \qquad (4.20)$$

where E is the field intensity.*

Conductors and insulators in an electric field. When a conductor is placed in an electric field, charges of unlike sign are induced in it. These charges are distributed over the surface of the conductor in such a manner that the intensity of the electrostatic field inside the conductor is zero, and the surface of the conductor is an equipotential surface.

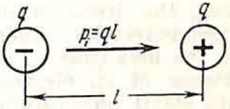


Fig. 38. Electric dipole

Insulators (dielectrics), when placed in an electric field, become *polarized*, i.e. the charges of the molecules are displaced so that their electric field resembles the field of two unlike point charges of equal magnitude (see Fig. 36*a*).

In general, a system of charges whose external field resembles the field of two unlike point charges of equal magnitude is called an *electric dipole* (Fig. 38).

The dipole is characterized by a vector quantity called the *electric dipole moment* (p_i):

$$p_i = ql \qquad (4.21)$$

where l is the distance between the charges.

---

* For an arbitrary field the concept "energy density at a point" is introduced:

$$w=\lim_{\Delta V\to 0}\frac{\Delta W}{\Delta V}$$

Here ΔW is the energy concentrated in the volume ΔV when the latter "contracts" to a point. If E is the intensity at this point, then formula (4.20) is also valid for an arbitrary field.

The direction of the vector p_i is taken from $-q$ to $+q$.

The magnitude of the total polarization of a dielectric is assessed by the *polarization vector*, equal to the vector sum of all the electric dipole moments in a unit volume:

$$P=\frac{1}{V}\sum p_i \tag{4.22}$$

The polarization and induction vectors are related as follows:

$$D = E + 4\pi P \text{ (in the CGSE system)}$$

$$D = \varepsilon_0 E + P \text{ (in the SI system)}$$

The molecules of some dielectrics are dipoles even in the absence of an electric field. The polarization of such substances consists in the alignment of the molecular dipoles along the direction of the field.

Ferroelectrics. Spontaneous polarization was detected for the first time in Seignette (Rochelle) salt. Even in the absence of an electric field ferroelectrics become divided into small (microscopic) regions that have an electric moment. These regions of spontaneous polarization are called *domains* (see also p. 174). The electric moments of domains in the absence of a field are aligned in different directions, and hence the electric moment of the entire ferroelectric will equal zero.

In an external electric field a ferroelectric becomes polarized as a whole owing to the change in the direction of polarization of the domains. After the action of the field has stopped, residual polarization remains.

The permittivity (dielectric constant) ε of ferroelectrics has great values (sometimes of the order of magnitude of several thousands). It depends very greatly on the intensity of the electric field.

At temperatures exceeding a certain value, called the *Curie point* (T_C), thermal motion destroys the domains, and the ferroelectric properties vanish.

The piezoelectric effect. Upon the mechanical deformation of some crystals along given directions electric charges of opposite signs appear on their faces, while inside the crystals an electric field arises. A change in the direction of the deformation causes a change in the signs of the charges. This phenomenon is called the *piezoelectric effect*. The piezoelectric effect is reversible, i.e. when a crystal is placed in an electric field, its linear dimensions change. The inverse piezoelectric effect is used to generate ultrasonic frequencies. The magnitude of the charge which arises in the piezoelectric effect is given by the relation

$$q = d_{11}F_x$$

where F_x is the force causing deformation, and d_{11}—a constant for the given crystal called the *piezoelectric modulus* (see Table 77).

TABLES AND GRAPHS

Table 72

Electric Field in the Earth's Atmosphere

Altitude, km . .	0	0.5	1.5	3	6	12
Intensity, V/m . .	130	50	30	20	10	2.5

Notes. 1. The charge of a thundercloud equals from 10 to 20 C (it may sometimes reach 300 C).

2. The mean surface density of the Earth's charge is 1.15×10^{-9} C/m². The total charge of the Earth is 5.7×10^5 C.

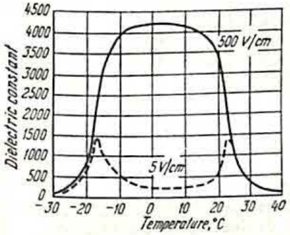


Fig. 39. Temperature dependence of dielectric constant of unsecured Seignette salt plate. The two curves correspond to two different field intensities

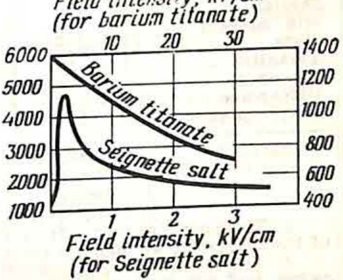


Fig. 40. Dependence of dielectric constant of barium titanate (scale at right) and Seignette salt (scale at left) on the field intensity (at 20° C)

Table 73

Electrical Insulating Materials

Material	ε	E_{pun}, kV/mm	d, g/cm³	ρ, $\Omega\cdot$cm
Amber	2.7-2.9	20-30	1.06-1.11	1×10^{18}
Asbestos	—	2	2.3-2.6	2×10^{5}
Bakelite (phenolic resin)	4-4.6	10-40	1.2	—
Beeswax	2.8-2.9	20-35	0.96	2×10^{10}-2×10^{15}
Birch, dry	3-4	40-60	0.7	—
Bitumen	2.6-3.3	6-15	1.2	—
Carbolite (P)	—	10-14.5	1.2-1.3	—
Celluloid	3-4	30	—	2×10^{10}
Ebonite (RP)	4-4.5	25	1.3	1×10^{18}
Eskapon (P)	2.7-3	36	—	—
Fibreboard, dry	2.5-8	2-6	1.1-1.94	5×10^{9}
Fluoroplastic-3	2.5-2.7	—	2.14	1.2×10^{18}
Getinax (laminated insulation) (P)	5-6.5	10-30	1.3	—
Glass	4-10	20-30	2.2-4.0	10^{11}-10^{14}
Gutta percha	4	15	0.95	2×10^{9}
Marble	8-10	6-10	2.7	1×10^{10}
Mica, muscovite	4.5-8	50-200	2.8-3.2	—
Ditto, phlogopite	4-5.5	60-125	2.5-2.7	10^{13}-10^{17}
Paraffin	2.2-2.3	20-30	0.4-0.9	3×10^{18}
Plexiglas	3.0-3.6	18.5	1.2	—
Polystyrene	2.2-2.8	25-50	1.05-1.65	5×10^{15}-5×10^{17}
Polyvinyl chloride	3.1-3.5	50	1.38	—
Porcelain, electrical	6.5	20	2.4	3×10^{14}
Pressboard	3-4	9-12	0.9-1.1	1×10^{9}
Radioporcelain (C)	6.0	15-20	2.5-2.6	—
Rosin	3.5	—	1.1	5×10^{16}
Rubber, soft	2.6-3	15-25	1.7-2.0	4×10^{13}
Shellac	3.5	50	1.02	1×10^{16}
Silk, natural	4-5	—	—	—
Slate	6-7	5-14	2.6-2.9	10^{8}
Textolite	7	2-8	1.3-1.4	—
Ticond (C)	25-80	15-20	3.8-3.9	—
Ultraporcelain (C)	6.3-7.5	15-30	2.6-2.9	3×10^{14}
Vinyl plastic (P)	4.1	15	—	—

Notes. 1. ε = permittivity (dielectric constant)
E_{pun} = puncture intensity
d = density
ρ = resistivity (see p. 137).

2. The puncture intensity is the maximum permissible value of the intensity; at higher values the dielectric is punctured.

3. The letters in parentheses stand for: P—plastic, C—ceramic, and RP—rubber plastic.

4. The values of the dielectric constant relate to temperatures from 18 to 20 °C. Its values for solids vary slightly with the temperature, except for ferroelectrics (see Fig. 39).

Table 74

Dielectric Constants (Permittivities) of Selected Pure Liquids

Substance	Temperature, °C						
	0	10	20	25	30	40	50
Acetone	23.3	22.5	21.4	20.9	20.5	19.5	18.7
Benzene	—	2.30	2.29	2.27	2.26	2.25	2.22
Carbon tetrachloride	—	—	2.24	2.23	—	2.20	2.18
Ethyl alcohol	27.88	26.41	25.00	24.25	23.52	22.16	20.87
Ethyl ether	4.80	4.58	4.38	4.27	4.15	—	—
Glycerine	—	—	56.2	—	—	—	—
Kerosene	—	—	2.0	—	—	—	—
Water	87.83	83.86	80.08	78.25	76.47	73.02	69.73

Note. Small quantities of impurities have a negligible effect on the value of the dielectric constant.

Table 75

Dielectric Constants (Permittivities) of Selected Gases (at 18 °C and Standard Pressure)

Substance	ε	Substance	ε
Air	1.00059	Nitrogen	1.00061
Carbon dioxide	1.00097	Oxygen	1.00055
Helium	1.00007	Water vapour	1.0078
Hydrogen	1.00026		

Note. The dielectric constants of gases decrease with an increase in the temperature, and increase with an increase in the pressure.

Table 76

Piezoelectric Moduli of Selected Crystals

Crystal	d_{11}, 10^{-8} (in CGSE units)	Crystal	d_{11}, 10^{-8} (in CGSE units)
Ammonium phosphate	148	Quartz	6.9
Barium titanate ceramic	750	Seignette satl	7000
Potassium phosphate	70	Tourmaline	5.78
		Zinc blende	9.8

Notes. 1. The moduli for some crystals vary depending on the direction of deformation; the greatest values are given in these cases.

2. To convert the value of a module from CGSE to SI units, multiply the figure in the table by 3×10^4. The module will then be expressed in C/N.

Table 77

Properties of Ferroelectric Crystals

Crystal	T_C, K	p_s (CGSE)	ε
$NaK(C_4H_4O_6) \cdot 4H_2O$ Seignette salt)	297-255	800	~ 9000
$NaK(C_4H_2D_2O_6) \cdot 4D_2O$	308-249	—	—
$LiNH_4(C_4H_4O_6) \cdot H_2O$	106	630	—
KH_2PO_4 (potassium dihydrophosphate)	123	16 000	~ 10^5
KD_2PO_4	218	18 000	—
KH_2AsO_4	96.5	—	—
$NH_4H_2PO_4$ (ammonium dihydrophosphate)	~ 398	—	90
$BaTiO_3$ (barium titanate)	391	48 000	~ 10^4
$KNbO_3$ (potassium niobate)	708	78 000	—
$NaNbO_3$	913	—	—
$LiTiO_3$	—	70 000	—

Notes. 1. T_C = Curie point

p_s = spontaneous polarization

ε = dielectric constant (permittivity).

2. The properties of some ferroelectrics manifest themselves within a definite temperature region. In these cases the upper and lower limits of the Curie point are given.

3. The maximum values of the dielectric constant are indicated.

4. To convert the values of the spontaneous polarization from CGSE to SI units, multiply the figures in the table by 3.3×10^{-12}.

B. DIRECT ELECTRIC CURRENT

FUNDAMENTAL CONCEPTS AND LAWS

1. Current in Metals

The ordered motion of charge carriers constitutes an *electric current*. In metals the charge carriers are electrons—negatively charged particles whose charge is equal to the elementary charge. The direction of the current is arbitrarily defined as the direction opposite to that in which the negative charges move.

If a charge Δq passes through a cross section of a conductor in a time from t to $t+\Delta t$, then the *current* at the instant t (or the *instantaneous current*) is the limit

$$i=\lim_{\Delta t\to 0}\frac{\Delta q}{\Delta t} \tag{4.23}$$

In *direct current* the same quantities of electricity (charges) pass through a cross section of a conductor in any equal intervals of time.

The unit of current in the SI system is the ampere (A). The definition of the ampere is given on p. 165. With a current of 1 A a charge of 1 C passes through the cross section of a conductor in 1 s.

The current passing through a unit cross-sectional area of a conductor is called the *current density* (j).

The unit of current density is the A/cm^2, i.e. a density at which a current of 1 A flows through an area of 1 cm^2 perpendicular to the direction of motion of the charges.

The current density is

$$j = nev \tag{4.24}$$

where n is the number of charge carriers in unit volume, e—the charge of a carrier, and v—the mean velocity of their ordered motion.

The *mobility of electrons* (u) numerically equals the mean velocity of their ordered motion (drift) acquired in a field having an intensity of 1 V/cm, i.e.

$$u=\frac{v}{E}$$

Hence

$$j = neuE = \sigma E \qquad (4.25)$$

where E is the intensity of the electric field inside a conductor, and σ—its conductivity (see below):

$$\sigma = neu \qquad (4.26)$$

Conductors in which the current is due to the motion of free electrons are called *electronic*. Metals are among these conductors.

If carriers of charges of different signs and magnitudes are present, the total current density will equal the sum of densities, each of which has been calculated for charges of a definite sign and magnitude:

$$j = \sum_i n_i e_i v_i \qquad (4.27)$$

The current is a scalar quantity, and the current density—a vector quantity.

To obtain a current in a conductor, a difference of potentials must be maintained on its ends. Devices making it possible to maintain a potential difference are called *current sources* (or *generators*). The terminals of the source by means of which it is connected to a consumer are called *poles*. The pole with the higher potential is called *positive*, and the other pole *negative*.

In current sources different kinds of energy not related to an electric field are converted into electric energy. A potential difference is maintained on the poles of an open source of current at the expense of the work of forces which in their nature differ from electric forces. Such forces are called *non-electromagnetic*, or *extraneous*. Extraneous forces acting inside a source carry the charges opposite to the direction of action of the electric forces; the latter transfer the charges inside the source from the positive pole to the negative one, and the extraneous forces transfer them from the negative pole to the positive one.

By the *electromotive force* (e.m.f.) *of a source* is meant a quantity numerically equal to the work of the extraneous forces in transferring a unit positive charge. The e.m.f. of a source is numerically equal to the potential difference across the ends of an open source.

The e.m.f. is measured in the same units as the voltage (for example in volts).

An e.m.f. appears upon the diffusion of ions in electrolytes (see p. 142), in electromagnetic induction (see p. 169), when semiconductor photocells are illuminated by light (see p. 210), etc.

An *electric circuit* consists of a current source, connecting wires, and instruments (or other devices) in which the current performs work (see Fig. 41). In the long run the work in a circuit is performed at the expense of the e.m.f. of the source.

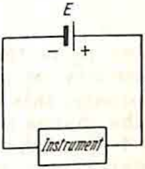


Fig. 41. Schematic view of an electric circuit

Ohm's law for a section of a circuit in which no extraneous forces act is as follows: the current in a conductor is proportional to the voltage across its ends, i.e.

$$i=\frac{U}{R} \tag{4.28}$$

The quantity $\frac{1}{R}$ in this relationship is a proportionality factor and is called the *conductance*. The quantity R is called the *resistance*.

The unit of resistance in the SI system is the ohm (Ω). 1 ohm is the resistance of a conductor in which a voltage of 1 V across its ends causes a current of 1 A to flow through it.

The resistance of a conductor (of constant cross section) is

$$R=\rho\frac{l}{S} \tag{4.29}$$

where ρ is the *resistivity*—the resistance of a conductor of unit length and unit cross-sectional area, l—the length of the conductor, and S—its cross-sectional area.

The quantity $\sigma=\frac{1}{\rho}$ is called the *conductivity*.

The unit of resistivity in the SI system is the ohm·m. In electrical engineering l is expressed in m, the cross-sectional area S—in mm^2; hence ρ is expressed in ohm·mm^2/m

$$1\ \text{ohm}\cdot\text{mm}^2/\text{m}=10^{-6}\ \text{ohm}\cdot\text{m}$$

The resistivity of most metals increases with the temperature.

The dependence of the resistivity on the temperature can be represented approximately by the relation

$$\rho_t = \rho_0 (1 + \alpha t) \tag{4.30}$$

where ρ_t is the resistivity at the temperature t, ρ_0—the resistivity at 0 °C, and α—the *temperature coefficient of resistivity*; this coefficient is numerically equal to the ratio of the change in resistivity caused by heating the conductor by 1 K to the initial resistivity. The resistivity of some metals at very low temperatures drops suddenly and becomes practically zero. This phenomenon is called *superconductivity*.

When resistors are connected in series the total resistance R_{ser} is equal to the sum of the separate resistances R_1, R_2, R_3, . . ., R_n:

$$R_{\text{ser}} = R_1 + R_2 + R_3 + \ldots + R_n \tag{4.31}$$

For resistors connected in parallel:

$$\frac{1}{R_{\text{par}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n} \tag{4.32}$$

Ohm's law for a section of a circuit in which an e.m.f. acts: for a section of a circuit containing an e.m.f. the following relation holds:

$$i = \frac{U + \mathscr{E}}{R} \tag{4.33}$$

where R is the resistance of the section, U—the voltage across the section, and $\mathscr{E}$—the e.m.f.

It should be borne in mind that both $\mathscr{E}$ and U may be positive or negative. The e.m.f. is considered *positive* if it increases the potential in the direction of current flow (the current flows from the negative to the positive terminal of the source); the voltage is considered *positive* if the current inside the source flows in the direction of decreasing potential (from the positive to the negative terminal). For example, in charging an accumulator (Fig. 42) the charging current

$$i_{\text{ch}} = \frac{U - \mathscr{E}_{\text{acc}}}{R_{\text{acc}}}$$

where U is the voltage across the terminals of the source in charging, $\mathscr{E}_{\text{acc}}$—the e.m.f. of the accumulator being charged, and R_{acc}—the resistance of the accumulator (the resistance of the connecting wires is disregarded).

For the section of the circuit ADB we have in the same case

$$i_{ch} = \frac{\mathscr{E}_{source} - U}{r}$$

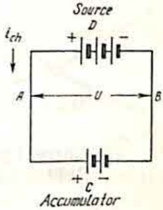


Fig. 42. Charging of an accumulator

where $\mathscr{E}_{source}$ is the e.m.f. of the source and r—its internal resistance.

For a closed unbranched circuit Eq. (4.33) becomes (in this case $U = 0$)

$$i = \frac{\mathscr{E}}{R + r} \tag{4.34}$$

where R is the external resistance of the circuit.

Work of an electric current. The work performed by a direct current in a section of a circuit is

$$A = iUt \tag{4.35}$$

where t is the duration of flow of the current, U—the voltage across the section, and i—the current.

The work of a current connected with a change in the internal energy of a conductor (the liberation of heat) in the absence of an e.m.f. across the section is

$$A = \frac{U^2}{R} t \tag{4.36}$$

The work connected with a change in the internal energy (regardless of whether or not there is an e.m.f. across the section) is

$$A = i^2RT \tag{4.37}$$

The unit of work (and also energy) in the SI system is the joule (J). 1 J is the work performed by a direct current of 1 A during 1 s on a section across which the voltage is 1 V. The unit kilowatt-hour (kW-h) is also used:

$$1 \text{ kW-h} = 3.6 \times 10^6 \text{ J}$$

Kirchhoff's laws. The calculation of currents, voltages, and e.m.f.'s in a branched circuit is performed on the basis of Kirchhoff's laws.

First law: the algebraic sum of the currents in sections of a circuit converging at any junction (branch point) equals zero. For example (Fig. 43)

$$i_1 + i_2 + i_3 - i_4 = 0$$

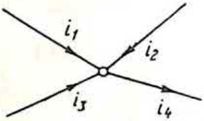


Fig. 43. Current junction

Second law: for any closed loop taken in a branched circuit the algebraic sum of the products of the currents and the respective resistances equals the algebraic sum of all the e.m.f.'s in the loop.

In summation, the currents whose directions coincide with the direction of reference around the loop, which is chosen arbitrarily, are assumed to be positive. E.m.f.'s are considered positive if they increase the potential in the direction of reference (i.e. this direction coincides with the passing from the negative to the positive pole). For example (Fig. 44)

$$i_1R_1 + i_2R_2 - i_3R_3 = \mathscr{E}_1 + \mathscr{E}_2 - \mathscr{E}_3$$

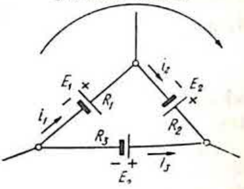


Fig. 44. Loop taken from a branched circuit

When identical sources are connected in series, we have

$$i\,(nr + R) = n\mathscr{E} \qquad (4.38)$$

where n is the number of sources, r—the internal resistance of one source, R—the external resistance, and $\mathscr{E}$ —the e.m.f. of one source.

When n identical sources are connected in parallel, we have

$$i\left(R + \frac{r}{n}\right) = \mathscr{E} \qquad (4.39)$$

2. Current in Electrolytes

Electrolytic conductors, or *electrolytes*, include solutions of acids, bases and salts in water or other solvents. Molten salts also conduct electricity. The charges in electrolytes are carried by *ions*—positively or negatively charged parts of molecules.

An electric field is created in an electrolyte between current-conducting plates immersed in it. These plates are called *electrodes*. The electrodes are connected to the poles of the e.m.f. source. The electrode connected to the positive pole is called an *anode*, and that connected to the negative pole a *cathode*.

The positive ions moving in the electric field towards the cathode are called *cations*; the negative ions moving towards the anode are called *anions*.

The current density created by ions of both signs is

$$j = n_+ev_+ + n_-ev_- \tag{4.40}$$

where n_+ and v_+ are the concentration and the drift velocity of the cations, respectively, e—the charge of an ion, and n_- and v_- —the concentration and drift velocity of the anions, respectively.

The mobility of ions is numerically equal to the mean drift velocity which an ion acquires in a field having the intensity 1 V/cm. The current density expressed through the ion mobilities u_+ and u_- is

$$j = (n_+u_+ + n_-u_-)\,eE \tag{4.41}$$

where E is the intensity of the electric field.

Ohm's law holds for electrolytes.

When a current flows through electrolytes (or molten salts), their chemical composition changes, and various products are deposited on or liberated at the electrodes. This phenomenon is called *electrolysis*.

Faraday's first law. The mass of a substance liberated at (deposited on) an electrode in electrolysis is proportional to the quantity of electricity Q passing through the electrolyte:

$$m = KQ \tag{4.42}$$

The proportionality factor K numerically equal to the mass of a given substance liberated when a unit quantity of electricity passes is called the *electrochemical equivalent* of this substance.

Faraday's second law. The electrochemical equivalent of a given substance is proportional to its chemical equivalent:

$$K = C\frac{A}{Z} \tag{4.43}$$

where $\frac{A}{Z}$ is the *chemical equivalent* equal to the ratio of the atomic weight of an element to its valence. The constant C

has the same value for all substances. The dimension of the chemical equivalent is g/g-equiv.

The Faraday constant. The same quantity of electricity approximately equal to 96 500 C (the *Faraday constant*) is needed to liberate in electrolysis an amount of any substance equal to its chemical equivalent $\frac{A}{Z}$:

$$F \approx 96\,500\ \text{C/g-equiv}, \qquad C = \frac{1}{F}\ \text{g-equiv/C}$$

Electrochemical cells. A potential difference sets in between an electrode immersed in an electrolyte and the solution. This difference is called the *electrochemical potential* of the given electrode in the given solution.

The values of the electrochemical potentials of metals in solutions with a normal concentration of their ions (i.e. with a concentration of 1 g-equiv of ions per litre) are called their *absolute normal (standard) potentials*. In these conditions the electrochemical potential depends only on the kind of metal.

When two electrodes are immersed in an electrolyte, a potential difference sets in between them equal to the difference between the electrochemical potentials of the electrodes. An electrolyte with two different electrodes immersed in it is called an *electrochemical*, or *galvanic cell* (for instance a copper and a zinc plates immersed in a solution of sulphuric acid form a so-called *Voltaic cell*).

Accumulators are electrochemical cells in which the electrodes are made of materials which restore their initial properties when a current is passed (charging) in the reverse direction compared with the current in discharging.

The quantity of electricity that can be obtained from an accumulator under given conditions of operation (temperature, discharging current, initial voltage) is called the *capacity of the accumulator*.

The capacity of an accumulator is expressed in ampere-hours (A-h):

$$1\ \text{A-h} = 3600\ \text{C}$$

3. Current in Gases

The passage of electric current through a gas is due to the presence of ions and free electrons. Electrons may become detached from neutral gas molecules and some of them may attach themselves to other neutral molecules and atoms. This process is called *ionization*. The work required to remove an electron from a molecule or atom is called the *ionization potential*.

The ionization potential is expressed in electron-volts (eV). 1 eV is equal to the energy acquired by an electron in passing through a field with a potential difference of 1 V.

The current density in gases, as in metals and liquids, is determined by the concentration of the charge carriers (ions), their mobility, and charge. However, in view of the fact that the ion concentration depends on the field intensity and varies throughout the volume of gas, Ohm's law does not apply, as a rule, to gaseous conductors.

Two kinds of conductivity are distinguished in gases: *induced conductivity*, when ionization is caused by agents other than an electric field (for example, X-rays, heating, etc.); and *intrinsic conductivity*, when ionization is due to the action of an electric field applied between the electrodes.

An electric current in a vacuum (for example, in thermionic tubes) is due to the motion of electrons or ions which escape from the electrodes placed in the vacuum.

To remove an electron from a metal, work must be performed; this is known as the *work function*.

When a metal is heated, it begins to emit electrons. This phenomenon is called *thermionic emission*. An electron can escape from the metal if the following condition is fulfilled:

$$\frac{1}{2} mv_{\text{n}}^2 \geqslant \varphi \tag{4.44}$$

where m is the mass of an electron, v_{n}—the projection of the thermal velocity of the electron onto a normal to the surface, and φ—the work function.

The maximum value of the thermionic emission current (at constant temperature) is called the *saturation current*. The density of the saturation current in thermionic emission is

$$j = A'T^2e^{-\varphi/kT} \tag{4.45}$$

where A' is a constant, T—the absolute temperature, k—the Boltzmann constant (see p. 78), and $e \approx 2.72$ is the base of natural logarithms. The quantities A' and φ are often called the *emission constants*.

According to theory, A' should be the same for all pure metals (60.2 $\text{A/cm}^2\cdot\text{K}^2$). Actually, however, it varies with different metals.

The so-called *oxide cathodes* have found wide application in practice. These cathodes are prepared by coating a metal base with barium oxide or an oxide of some other metal, which considerably decreases the work function.

A discharge in the form of a spark (breakdown) occurs between cold electrodes in a gas at great field intensities. The voltage needed to cause breakdown depends on the material, shape, and dimensions of the electrodes, on the distance between them, and also on the properties and pressure of the gas.

For flat parallel electrodes whose dimensions are compatible with the distance between them, the breakdown voltage for a given gas and electrode material depends only on the product pd (where p is the pressure of the gas, and d—the distance between the electrodes). If p and d are varied so that their product remains constant, the value of the breakdown voltage will not change.

The distance between electrodes at which breakdown occurs at a given voltage is called the *spark gap*. The length of a spark gap allows us to determine the voltage between electrodes.

4. Semiconductors

Semiconductors are substances whose electrical conductivity is due to the motion of electrons and whose resistivity at room temperature lies within the range of from 10^{-2} to 10^{9} ohm·cm. The resistivity of semiconductors is strongly temperature-dependent. In contrast to metals the resistivity of semiconductors decreases with an increase in the temperature. The resistivity of semiconductors depends strongly on the presence of impurities.

The electrons in an atom are at discrete energy levels (see p. 228); each electron has a quite definite value of its energy, which differs from the energy of the other electrons. An isolated atom cannot have more than two electrons on the same energy level; but these electrons will also differ from each other in the orientation of their spins (see p. 229). This law is called *Pauli's principle*.

In isolated atoms of a substance all the energy levels are the same. Upon reaction (interaction), the energy levels of each atom change slightly in comparison with the energy levels of unreacted atoms; the energy levels of reacting atoms will differ from one another.

Figure 45*a* shows as an example the energy levels K and L of unreacted atoms; upon the reaction of n atoms each of the levels "decomposes" into n different levels shown in Fig. 45*b*. The difference between the energies of the "decomposed" levels is about 10^{-22} to 10^{-23} eV. The complex of decomposed energy levels forms a band of permitted (allowed) energy levels. The bands are separated from one another by intervals of forbidden energy values; these energy intervals are called *forbidden bands* (or *gaps*); an electron cannot have energy levels that are within a forbidden band.

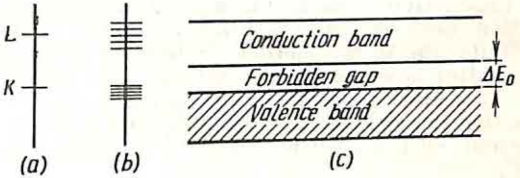


Fig. 45. Electron energy levels in semiconductors

The electrical conductivity of both metals and semiconductors is due only to the valence electrons because the electrons of the inner shells are firmly bound to the nucleus. At 0 K the valence electrons occupy the lowest energy levels. All the allowed levels in this band are occupied. This band is called the *filled* (or *valence*) *band*. The second band of allowed energy levels does not contain a single electron at 0 K; this band is called a *conduction band*. The filled band and the conduction band are separated by a forbidden gap (Fig. 45*c*). The energy needed for an electron to transfer from the filled band to the conduction band is called the *width of the forbidden gap* (ΔE_0). In metals the conduction and valence bands overlap; in insulators $\Delta E_0 > 2$ eV.

The appearance of electrical conductivity is connected with the presence of electrons in the conduction band; if there are no electrons in this band, there is no conductivity.

Thermal motion (apart from other action) ensures transitions of electrons to the conduction band. The number of electrons in the conduction band is determined by the expression

$$n = Ae^{-\Delta E_0/2kT} \tag{4.46}$$

where A is a constant, k—the Boltzmann constant, and T—the absolute temperature.

The conductivity is

$$\sigma \approx \sigma_0 e^{-\Delta E_0/2kT} \tag{4.47}$$

After an electron has transferred to the conduction band vacant levels appear in the valence band. In the presence of an external electric field the electrons will migrate in both bands. Conductivity due to the motion of electrons in the conduction band is called *electronic* (*n-type*) *conductivity*; conductivity due to the motion of electrons in the valence band is called *hole* (*p-type*) *conductivity*. The motion of an electron in a filled zone can be considered as the movement of a positive charge in the direction opposite to the motion of the electron. Such a positive charge is conventionally called a "hole".

Conductivity due to the motion of an identical number of electrons and holes, which are formed as a result of migration of electrons from the valence band to the conduction band, is called intrinsic. Intrinsic conductivity appears as a result of violation of valence bonds.

In the practical application of semiconductors the greatest significance is acquired by *impurity conductivity*, which is determined by impurities of other substances. There are two kinds of impurities—donor and acceptor. *Donor impurities* create additional allowed energy levels near the upper boundary of the forbidden band; the atoms of donor impurities give up their electrons to the conduction band and thus ensure *impurity electronic conductivity*. *Acceptor impurities* create additional levels near the lower boundary of the forbidden band; the atoms of acceptor impurities pick up electrons from the valence band on their levels and thus ensure *impurity hole conductivity*.

In germanium, impurities of elements of Group V of the Periodic Table (for example, antimony) are donor, and impurities of Group III elements (for example, gallium) are acceptor.

Impurity conductivity is possible when both acceptor and donor impurities have been introduced into a semiconductor.

It should be noted that all semiconductors always have electrons and holes, but their contribution to the electrical conductivity may vary because of different concentrations or mobilities.

5. Thermoelectricity

If a closed circuit is composed of two dissimilar conductors and their junctions are kept at different temperatures, a current will flow through the circuit. This current will be maintained by the e.m.f. appearing at the junctions. The e.m.f. appearing in these conditions is called the *thermal e.m.f.*, and the phenomenon itself is called the *thermoelectric effect*, or *thermoelectricity*.

Within a certain temperature range the magnitude of the thermal e.m.f. is approximately proportional to the temperature difference. In this case

$$\mathscr{E}_{\mathrm{th}} = \alpha (T_1 - T_2)$$

The quantity α is called the *differential thermal e.m.f.* (or the *coefficient of the thermal e.m.f.*); it is numerically equal to the thermal e.m.f. developed per kelvin.

TABLES AND GRAPHS

Electric Currents in the Earth's Atmosphere

The electric field of the Earth's atmosphere (see Table 72) causes a stream of ions to appear in the atmosphere, i.e. a conduction current directed vertically downward. The density of this current does not practically change with the altitude and in a region with clear weather is $2\text{-}3 \times 10^{-16}$ A/cm². Currents of the opposite direction appear in regions with thunderstorms.

The current density in the hydrosphere is 10^{-6} A/cm².

The density of the currents due to the motion of charges on raindrops, snowflakes, and hail is:

for light rain—10^{-11} to 10^{-10} A/cm²,

for thunderstorms and hail—up to 10^{-8} A/cm².

The current in a lightning stroke reaches 500 000 A (the most frequent values range from 20 000 to 40 000 A).

The voltage in a lightning stroke reaches 10^9 V; the discharge lasts about 10^{-3} s, its length is about 10 km, and the thickness of the channel is up to 20 cm.

Table 78

Resistivity and Temperature Coefficient of Resistivity of Metals (at 20 °C)

Metal	ρ, ohm·mm²/m	α, K⁻¹
Aluminium	0.028	0.0049
Brass	0.025-0.06	0.002-0.007
Chromium	0.027	—
Copper	0.0175	0.0039
Iron	0.098	0.0062
Lead	0.221	0.0041
Mercury	0.958	0.0009
Molybdenum	0.057	0.0033
Nickel	0.100	0.0050
Phosphor bronze	0.08	0.0040
Silver	0.016	0.0036
Tantalum	0.155	0.0031
Tin	0.115	0.0042
Tungsten	0.055	0.0045
Zinc	0.059	0.0035

Note. The values given in the table are average; for different samples they depend on the degree of purity, thermal treatment, etc.

The temperature coefficient of resistivity of pure metals is close to $\frac{1}{273} = 0.00367\ \mathrm{K}^{-1}$.

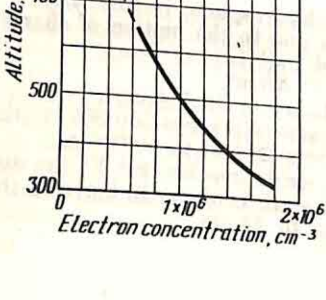


Fig. 46. Change in concentration of electrons in atmosphere with altitude (determined by measurements made on satellites and rockets). The dash line shows assumed values

Table 79

Transition Temperature of Selected Metals and Alloys to the Superconducting State

Substance	T, K
Metals	
Zirconium	0.3
Cadmium	0.6
Zinc	0.8
Aluminium	1.2
Tin	3.7
Mercury	4.1
Tantalum	4.4
Lead	7.3
Niobium	9.2
Alloys	
Bi-Pt	0.16
Pb-Au	2.0-7.3
Sn-Zn	3.7
Pb-Hg	4.1-7.3
Sn-Hg	4.2
Pb-Ag	5.8-7.3
Pb-Sb	6.6
Pb-Ca	7.0
Compounds	
NiBi	4.2
PbSe	5.0
$NbBi_2$	5.5
NbB	6
MoC	7.6-8.3
Nb_2C	9.2
NbC	10.1-10.5
NbN	15-16
V_3Si	17.1
Nb_3Sn	18

Notes. 1. There are a number of superconducting alloys containing a greater number of components: Rose's metal (8.5 K), Newton's metal (8.5 K), Wood's metal (8.2 K), Pb-As-Bi (9.0 K), Pb-As-Bi-Sb (9.0 K).

2. Upon transition to the superconducting state the resistivity of compounds and alloys varies throughout a wide range of temperatures (sometimes about 2 K). In addition the transition temperature depends on the heat treatment of the alloy or compound. In such cases the table indicates the limits within which the transition temperature lies.

Table 80

Alloys of High Ohmic Resistance (at 20 °C)

Alloy	ρ, ohm·mm²/m	α, K⁻¹	t_{max}, °C
Constantan (58.8% Cu, 40% Ni, 1.2% Mn)	0.44-0.52	0.00001	500
Fechral (80% Fe, 14% Cr, 6% Al)	1.1-1.3	0.0001	900
German silver (65% Cu, 20% Zn, 15% Ni)	0.28-0.35	0.00004	150-200
Manganin (85% Cu, 12% Mn, 3% Ni)	0.42-0.48	0.00003	100
Nichrome (67.5% Ni, 15% Cr, 16% Fe, 1.5% Mn)	1.0-1.1	0.0002	1000
Nickeline (54% Cu, 20% Zn, 26% Ni)	0.39-0.45	0.00002	150-200
Rheotan (84% Cu, 12% Mn, 4% Zn)	0.45-0.52	0.0004	150-200

Notes. The mean values of the temperature coefficient of resistance α are indicated for the temperature interval from 0 to 100 °C. The last column indicates the maximum permissible working temperatures.

The values of the temperature coefficient of resistance of constantan varies from −0.00004 to +0.00001 depending on the sample.

Table 81

Allowed Currents in Insulated Conductors for Prolonged Operation, A

Material \ Cross-sectional area, mm²	1	1.5	2.5	4	6	10	16	25
Copper	11	14	20	25	31	43	75	100
Aluminium	8	11	16	20	24	34	60	80
Iron	—	—	8	10	12	17	30	—

Table 82

Fuse Links

Current, A	5	15	30	60	100
Diameter of tinned copper wire, mm	0.213	0.508	0.914	1.42	2.03

Note. The rated current indicated on a fuse is the maximum current that may flow for a long time. A current from 1.8 to 2 times greater than the nominal one will quickly melt the wire in the fuse link.

Table 83

Resistivity of Electrolytes at Different Concentrations p' (at 18 °C)

Solute	p', %	d, g/cm^3	ρ, ohm·cm	$\varkappa$, K^{-1}
Ammonium chloride	5 10 20	1.011 1.029 1.057	10.9 5.6 3.8	0.0198 0.0186 0.0161
Copper sulphate	5 10 17.5	1.062 1.107 1.206	52.9 31.5 23.8	0.0216 0.0218 0.0236
Hydrochloric acid	5 20 40	1.023 1.1 1.2	2.5 1.3 1.9	0.0158 0.0154 —
Nitric acid	10 20 30 40	1.05 1.12 1.18 1.25	2.1 1.5 1.3 1.4	0.0145 0.0137 0.0139 0.0150
Sodium chloride (table salt)	5 10 20	1.034 1.071 1.148	14.9 8.3 5.1	0.0217 0.0214 0.0716
Sodium hydroxide	5 10 20 40	1.05 1.11 1.22 1.43	5.1 3.2 3.0 8.3	0.0201 0.0217 0.0299 0.0648
Sulphuric acid	5 20 30 40	1.032 1.14 1.22 1.30	4.8 1.5 1.4 1.5	0.0121 0.0145 0.0162 0.0178
Zinc sulphate	5 10 20	1.062 1.107 1.232	52.4 31.2 21.3	0.0225 0.0223 0.0243

Note. The resistivity of electrolytes diminishes with increasing temperature (in contrast to metals). The resistivity ρ_t for other temperatures can be calculated by the formula [compare with Eq. (4.30)] $\rho_t = \rho_{18}\,[1 - \varkappa\,(t - 18\ °\mathrm{C})]$, where $\varkappa$ is the temperature coefficient given in the table, ρ_{18}—the resistivity at 18 °C, t—the temperature for which the resistivity ρ_t is being found, and d—the density.

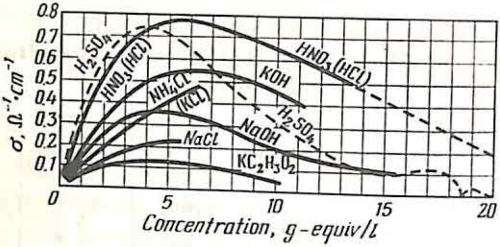


Fig. 47. Dependence of electrical conductivity on concentration of aqueous solutions of selected compounds (at 18 °C)

Table 84

Thermal E.M.F. of Selected Metal Couples, mV

Junction temperature, °C	Platinum-platinum with 10% rhodium	Iron-constantan	Copper-constantan	Junction temperature, °C	Platinum-platinum with 10% rhodium	Iron-constantan	Copper-constantan
−200	—	8	5.5	600	5.22	33	—
100	0.64	5	4	700	6.26	39	—
200	1.44	11	9	800	7.33	46	—
300	2.32	16	15	1000	9.57	58	—
400	3.25	22	21	1500	15.50	—	—
500	4.22	27	—				

Note. The temperature of the reference junction is kept at 0 °C.

Table 85

Differential Thermal E.M.F. (α) Relative to Platinum (at 0 °C)

Metal or compound	α, μV/K	Metal or compound	α, μV/K
Antimony . . .	17.0	Iron	16.0
Bismuth	−65.0	Lead telluride	−300
Constantan . .	−34.4	Nickel	−16.4
Copper	7.4	Zinc antimonide	200
Cuprous oxide .	1000		

Note. The minus signs indicate that the current in the hot junction flows from the metal with the smaller algebraic value of α. For example, in the thermocouple copper-constantan (Fig. 48) the current in the hot junction flows from constantan to copper.

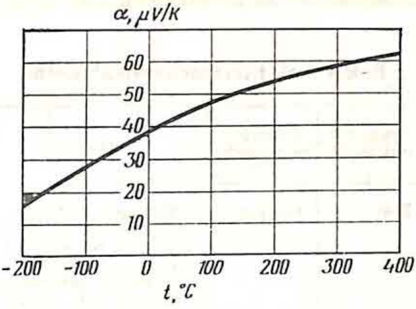


Fig. 48. Temperature dependence of differential thermal e.m.f. for a copper-constantan thermocouple

Table 86

Electrochemical Equivalents

Ion	g-equiv	K, mg/C	Ion	g-equiv	K, mg/C
H^+	1.008	0.0104	CO_3^{2-}	30.0	0.3108
O^{2-}	8.0	0.0829	Cu^{2+}	31.8	0.3297
Al^{3+}	9.0	0.0936	Zn^{2+}	32.7	0.3387
OH^-	17.0	0.1762	Cl^-	35.5	0.3672
Fe^{3+}	18.6	0.1930	SO_4^{2-}	48.0	0.4975
Ca^{2+}	20.1	0.2077	NO_3^-	62.0	0.642
Na^+	23.0	0.2388	Cu^+	63.6	0.6590
Fe^{2+}	27.8	0.2895	Ag^+	107.9	1.118

Note. The number of plus or minus signs in the superscript denotes the number of elementary charges carried by one ion.

Table 87

Absolute Standard (Normal) Potentials of Selected Metals

Metal	U_{stan}, V	Metal	U_{stan}, V
Cadmium	−0.13	Manganese	−0.78
Chromium	−0.29	Mercury	1.13
Copper	0.61	Nickel	0.04
Iron	−0.17	Silver	1.07
Lead	0.15	Zinc	−0.50

Table 88

E.M.F. of Electrochemical Cells

Name of cell	Negative electrode	Positive electrode	Solution	e.m.f., V
Daniell	Zinc	Copper	Different solutions at electrodes: zinc solution of sulphuric acid (5-10%); copper in saturated solution of copper sulphate $CuSO_4$	1.1
Edison (Ni-Fe) accumulator	Powdered iron (or cadmium mixed with iron oxides)	Nickel dioxide	20% solution of potassium hydroxide (KOH)	1.4-1.1
Grenet	Zinc	Carbon	12 parts $K_2Cr_2O_7$, 25 parts H_2SO_4, 100 parts H_2O	2.01
Lead accumulator	Spongy lead	Lead peroxide PbO_2	27-28% solution of H_2SO_4, free from chlorine, density 1.20	2.0-1.9 at 15 °C
Leclanché	Zinc	Carbon	Solution of ammonium chloride, manganese peroxide with powdered carbon	1.46
Leclanché (dry)	Zinc	Carbon	1 part ZnO, 1 part NH_4Cl, 3 parts $ZnCl_2$ and water until a paste is formed	1.3
Silver-zinc accumulator	Zinc oxide	Silver	Solution of potassium hydroxide (KOH)	1.5
Weston (standard)	Cadmium amalgam	Mercury	Saturated solution of $CdSO_4$, paste of Hg_2SO_4 and $CdSO_4$	1.0183

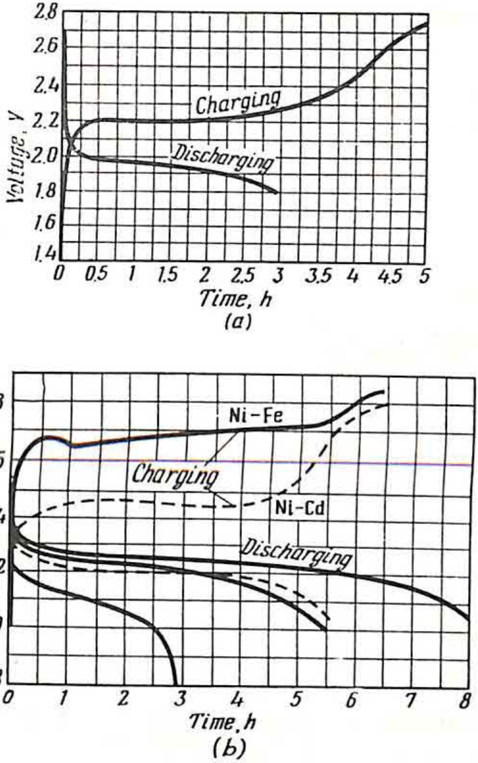


Fig. 49: (*a*) Change in voltage across one cell of an acid accumulator in charging with a normal current of $\frac{Q}{4}$ A (Q is the capacity of the accumulator in A-h) and in discharging with a current for three-hour duty $\left(\frac{Q}{3}\text{ A}\right)$; (*b*) change in voltage across one cell in charging and discharging of nickel-acid (solid lines) and nickel-cadmium (dash line) accumulators. Charging is in normal duty of $\frac{Q}{6}$ A (6 h), discharging is in five-hour duty $\left(\frac{Q}{5}\text{ A}\right)$. For nickel-iron accumulators the curves are for eight-hour $\left(\frac{Q}{8}\text{ A}\right)$ and three-hour $\left(\frac{Q}{3}\text{ A}\right)$ discharging duties

Table 89

Mobility of Ions in Aqueous Solutions (at 18 °C)

Cation	cm²/s · V	Anion	cm²/s · V
H^+	0.003263	OH^-	0.00180
K^+	0.000669	Cl^-	0.00068
Na^+	0.000450	NO_3^-	0.00062
Ag^+	0.00056	SO_4^{2-}	0.00068
Zn^{2+}	0.00048	CO_3^{2-}	0.00062
Fe^{3+}	0.00046		

Notes. 1. The mobility of ions grows by about 2% when the temperature increases by 1 K.

2. The number of plus or minus signs in the superscript indicates the number of elementary charges carried by one ion.

Table 90

Mobility of Electrons in Metals (in cm²/s · V)

Metal	Ag	Na	Be	Cu	Au	Li	Al	Cd	Zn
Mobility . . .	56	48	44	35	30	19	10	7.9	5.8

Note. The field intensity inside metals actually does not exceed 0.001 V/cm; hence, the electron velocities will be numerically much smaller than the values of the mobility given in the table. This can easily be verified by means of the relation (4.24) by inserting the permissible values of the current density given in Table 81.

Table 91

Mobility of Ions in Gases (at 760 mm Hg and 20 °C, cm²/s·V)

Gas	Positive ion	Negative ion
Air, dry	1.4	1.9
Ditto, saturated with water vapour	1.4	2.1
Argon	1.5	1.7
Carbon dioxide	0.8	0.8
Helium	16.0	—
Hydrogen	6.3	8.1
Mercury vapour (at pressure of 1 mm Hg)	220	—
Nitrogen	2.7	—
Oxygen	1.3	1.8

Notes. 1. In the general case the mobility depends on the ratio of the electric field intensity in the gas E to the gas pressure p. At low values of $\frac{E}{p}$ the mobility remains unchanged; the mobility of ions changes at values of their drift velocities comparable with the velocities of their thermal motion.

2. The mobility of a given species of ions changes inversely proportional to the density of the gas; at a constant temperature the mobility changes inversely proportional to the pressure (within the pressure interval from 0.1 mm Hg to 60 atm). The mobility only slightly depends on the charge of an ion.

3. The mobility greatly depends on the purity of a gas. Therefore the values given in the table should be considered as approximate.

Table 92

Ionization Potentials

Ionization	E_{ion}, eV	Ionization	E_{ion}, eV
$He \longrightarrow He^+$	24.5	$H \longrightarrow H^+$	13.5
$Ne \longrightarrow Ne^+$	21.5	$O \longrightarrow O^+$	13.5
$N_2 \longrightarrow N_2^+$	15.8	$H_2O \longrightarrow H_2O^+$	13.2
$Ar \longrightarrow Ar^+$	15.7	$Xe \longrightarrow Xe^+$	12.8
$H_2 \longrightarrow H_2^+$	15.4	$O_2 \longrightarrow O_2^+$	12.5
$N \longrightarrow N^+$	14.5	$Hg \longrightarrow Hg^+$	10.4
$CO_2 \longrightarrow CO_2^+$	14.4	$Na \longrightarrow Na^+$	5.1
$Kr \longrightarrow Kr^+$	13.9	$K \longrightarrow K^+$	4.3

Table 93

Emission Constants of Selected Metals and Semiconductors

Element	φ, eV	A', $A/cm^2 \cdot K^2$	Element	φ, eV	A', $A/cm^2 \cdot K^2$
Aluminium	3.74	—	Nickel	4.84	30
Antimony	2.35	—	Platinum	5.29	32
Barium	2.29	—	Selenium	4.72	—
Cesium	1.89	160	Silicon	4.10	—
Chromium	4.51	48	Tellurium	4.12	—
Copper	4.47	65	Thorium	3.41	70
Germanium	4.56	—	Tin	4.31	—
Iron	4.36	26	Tungsten	4.50	60-100
Molybdenum	4.37	115	Uranium	3.74	—

Note. The work function depends markedly on the cleanliness of the surface and on impurities. The figures in the table are for pure specimens.

Table 94

Emission Constants of Films on Metals

Metal	Film	φ, eV	A', $A/cm^2 \cdot K^2$
Molybdenum	Thorium	2.58	1.5
Tantalum	Ditto	2.52	0.5
Tungsten	Barium	1.56	1.5
Ditto	Cesium	1.36	3.2
Ditto	Thorium	2.63	3.0
Ditto	Uranium	2.81	3.2
Ditto	Zirconium	3.14	5.0

Table 95

Emission Constants of Oxide-Coated Cathodes

Cathode	φ, eV	A', $A/cm^2 \cdot K^2$
Barium on oxidized tungsten	1.10	0.3
Nickel-BaO-SrO	1.20	0.96
Barium-oxygen-tungsten	1.34	0.18
Pt-Ni; BaO-SrO	1.37	2.45
BaO on a nickel alloy	1.50-1.83	0.087-2.18
Thorium oxide-coated cathode (mean value)	2.59	4.35

Table 96

Properties of Semiconductors

Substance	t_m, °C	ΔE_0, eV	u_-, cm^2/s·V	u_+, cm^2/s·V
Boron (B)	2300	1.1	10	10
Graphite (C)	—	0.1	—	—
Diamond (C)	—	6-7	1800	1200
Silicon (Si)	1414	1.12	1900	500
Germanium (Ge)	958	0.75	3900	1900
Tin (gray) (Sn)	—	0.08	3000	—
Sulphur (S)	113	2.4	—	—
Selenium (gray) (Se)	220	2.3	—	—
Tellurium (Te)	452	0.36	1700	1200
Iodine (I)	113.5	1.3	25	—
Ag_2Te	955	0.17	4000	—
HgTe	670	0.2	10000	100
B_2Te_3	585	0.25	600	150
Mg_2Sn	778	0.36	200	150
PbSe	1065	0.5	1400	1400
ZnTe	1240	0.6	100	—
PbS	1114	1.2	650	800
AgBr	430	1.35	35	—
CdTe	1045	1.45	450	100
Cu_2O	1232	1.5-1.8	—	100
Al_2O_3	2050	2.5	—	—
ZnO	1975	3.2	200	—

Notes. 1. t_m is the melting point, ΔE_0—the width of the forbidden band, and u_- and u_+—the mobility of electrons and holes, respectively.

2. The values of the mobilities are given for room temperature and field intensities less than the critical field.

3. Deviations from Ohm's law may occur owing to the dependence of the mobility on the intensity of the electric field in semiconductors The lowest intensity beginning from which such deviations are observed is called the *critical field* (E_{cr}). The values of E_{cr} at 20 °C are:

n-type germanium—900 V/cm
p-type germanium—1400 V/cm
n-type silicon—2500 V/cm
p-type silicon—7500 V/cm

The critical field diminishes with decreasing temperature.

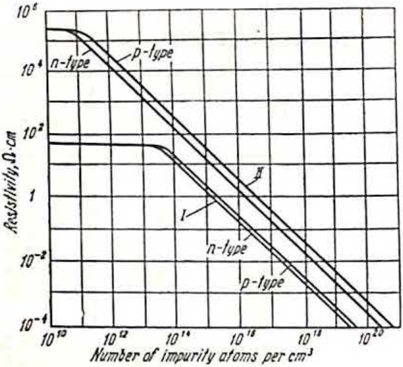


Fig. 50. Dependence of resistivity of germanium (*I*) and silicon (*II*) on impurity atom concentration. The temperature is about 20 °C

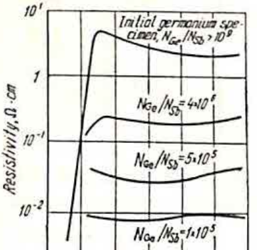


Fig. 51. Temperature dependence of resistance of germanium.

The values of the resistivity are plotted on the axis of ordinates to a logarithmic scale, the reciprocal of the absolute temperature is plotted on the axis of abscissas; N_{Ge} is the number of germanium, and N_{Sb}—the number of antimony atoms

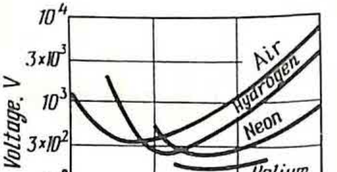


Fig. 52. Dependence of breakdown voltage for flat metal electrodes on value of *pd* (*p* is the pressure of the gas, and *d*—the distance between electrodes)

Table 97

Spark Gaps for Air
(mm at pressure of 760 mm Hg)

Shape of metal electrodes Voltage, V	Two points	Two spheres 5 mm in diameter	Two plates
20 000	15.5	5.8	6.1
40 000	45.5	13	13.7
100 000	200	45	36.7
200 000	410	262	75.3
300 000	600	530	114

C. ELECTROMAGNETISM

FUNDAMENTAL CONCEPTS AND LAWS

1. Induction of Magnetic Field. Interaction of Currents. Magnetic Moment

Two conductors with currents, a conductor with current and a magnet, or two magnets interact with each other. A field called *magnetic* is responsible for this interaction. A magnetic field appears as a result of charge drift, the magnetic field of magnets is created by the drift of electrons in atoms. No magnetic field appears around charges at rest.

A magnetic field is detected according to its action on conductors with current (or drifting charges) and magnetic needles; the forces causing this action are called *magnetic*; no magnetic field acts on charges at rest.

The vector quantity B, called the *magnetic induction*, is introduced to characterize a magnetic field. The direction of the magnetic induction vector coincides with the direction of the force acting on the north end of a magnetic needle placed at the given point of the field. The magnitude of the force acting on a conductor with a current in a magnetic field is determined by *Ampere's law*:

$$\Delta F = ki\, \Delta l B \sin \beta \qquad (4.48)$$

where i is the current, Δl—a small (elementary) length of the conductor (an element of the conductor length), B—the magnetic induction, β—the angle between B and Δl; the element of the conductor length Δl is a vector whose direction coincides with that of the current; the product $i\Delta l$ is called a *current element*. The proportionality factor k depends on the choice of the units; when all the quantities are measured in one system of units, $k = 1$.

The magnetic induction is numerically equal to the force with which a magnetic field acts on a unit current element ($i\Delta l = 1$) arranged at right angles to the induction vector.

The magnetic induction depends on the properties of the medium.

In the SI system the unit used to measure magnetic induction is the tesla (T). 1 tesla is the induction of a field that acts on a unit current element $i\Delta l = 1$ A·m arranged at right angles to the induction vector with a force of 1 N.

The CGSM system of units (see p. 164), in which the unit of current is equal to 10 amperes, is often used for measuring quantities characterizing a magnetic field. In this system the unit of magnetic induction is the gauss (Gs). 1 gauss is the induction of a field that acts on a unit current element $i\Delta l = 10$ A·cm arranged at right angles to the induction vector with a force of 1 dyne;

$$1 \text{ T} = 10^4 \text{ Gs}$$

A quantity, called the *magnetic field intensity* (or simply the *magnetic intensity*), is introduced to characterize a magnetic field in a vacuum. The magnetic field intensity H is numerically equal to the induction of a magnetic field in a vacuum.

The dimensions of B and H in the CGSM system are the same, whereas in the SI system they differ from each other.

To determine the magnetic field intensity, we must remove the substance from the space in which there is a field, and then measure the force acting on a unit current element ($i\Delta l = 1$) arranged at right angles to the induction vector.

The magnetic field intensity does not depend on the properties of the medium, but is determined only by the current and the shape of the conductor.

The ratio

$$\frac{B}{H}=\mu_a$$

is called the *absolute permeability* of a medium.

The magnetic field intensity is a vector quantity coinciding (in a homogeneous medium) in direction with the magnetic induction vector, but whose value is μ_a times smaller than the latter.

The numerical value of μ_a is expressed in relative units (relative to the absolute value of the permeability of a vacuum μ_0). The quantity

$$\mu=\frac{\mu_a}{\mu_0}$$

is called the *relative permeability* (or simply the *permeability*). It does not depend on the system of units used.

The direction of the force acting on a current-carrying conductor is determined by means of the *left-hand rule*: if the open palm of the left hand is placed so that the lines of force of the magnetic field enter the palm, while the outstretched fingers point in the direction of the current, then the thumb will indicate the direction of the force acting on the conductor (Fig. 53).

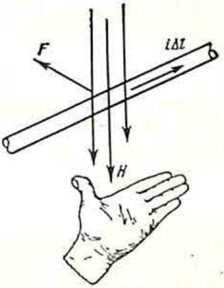


Fig. 53. Left-hand rule

Two sufficiently long straight and parallel conductors with current interact with each other so that if the currents have the same direction, they are attracted, and if the currents have opposite directions, they are repulsed from each other.

The mathematical expression of this law is:

$$F=\frac{2\mu i_1 i_2}{a}\, l \text{ (CGSM)} \tag{4.49}$$

$$F=\frac{\mu\mu_0 i_1 i_2}{2\pi a}\, l \text{ (SI)} \tag{4.50}$$

where a is the distance between the conductors, l—the length of the conductors, i_1 and i_2—the currents in them, μ—the permeability of the medium.

The force acting on a moving charge in a magnetic field (called the *Lorentz force*) is

$$F_L = qvB \sin \alpha \tag{4.51}$$

where q is the charge of a particle, v—the velocity, and α—the angle between the direction of the velocity and the induction B. The Lorentz force is directed perpendicularly to the plane in which the vectors B and v are.

The moment of the forces acting on a plane current loop placed in a magnetic field is

$$M = iSB \sin \alpha \tag{4.52}$$

where i is the current, S—the area of the loop, B—the magnetic induction, and α—the angle between a perpendicular to the plane of the loop and the vector B.

The quantity

$$p_m = iS$$

is called the *magnetic moment* of the loop.

The magnetic moment is a vector quantity. The direction of the magnetic moment is determined according to the *right-hand screw rule*: if the head of a screw is rotated in the direction of the current in a loop, the forward motion of the screw will coincide with the direction of p_m.

The magnetic moment of several current loops equals the vector sum of their magnetic moments.

The magnetic moment of a particle (with the charge q) travelling along a circular orbit with the radius R and the linear velocity v is

$$p_m = \frac{1}{2} qvR \tag{4.53}$$

2. CGSM and SI Systems of Units

In the CGS electromagnetic (CGSM) system the basic units are the centimetre, gramme (mass), second, and for electric quantities—the permeability of a vacuum ($\mu_0 = 1$). The unit of current in this system is derived and is determined from the law of interaction of currents.

The unit of current in the CGSM system is such a direct current which, when flowing through two infinitely long parallel conductors placed in a vacuum 1 cm apart, causes them to interact with a force of 2 dynes per cm of their length. It is assumed that both conductors have a sufficiently small cross-sectional area.

The basic units of the SI system are the metre, kilogram (mass), second, and the unit of current—ampere.

An ampere is such a direct current which, when flowing through two thin infinitely long parallel conductors placed in a vacuum at a distance of one metre from each other, causes them to interact with a force of 2×10^{-7} N per metre of their length.

In this system the permeability is a derived quantity. For a vacuum

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H/m} = 1.26 \times 10^{-6}\ \mathrm{H/m}$$

3. Intensity of the Magnetic Fields of Currents

The *lines of force* (*force lines*) of a magnetic field are lines the tangents to which coincide with the direction of the intensity at the given point. The magnetic lines of force are closed (in contrast to the force lines of an electrostatic field); such fields are called *vortex* (Figs. 54-55). The force lines of a straight conductor with current are concentric circles in a plane perpendicular to the conductor (Fig. 56). The direction of a force line of a magnetic field is determined by the right-hand screw rule: if a screw is turned so that it moves forward in the direction of the current, the direction of rotation of its head will coincide with that of the force lines.

The intensity of the magnetic field induced by a current element $i\Delta l$ in the CGSM system (see Fig. 56) is

$$\Delta H = \frac{i\Delta l \sin\alpha}{r^2} \qquad (4.54)$$

and in the SI system is

$$\Delta H = \frac{i\Delta l \sin\alpha}{4\pi r^2} \qquad (4.55)$$

where r is the distance from the current element to the point at which the intensity is being determined, and α—the angle between r and $i\Delta l$. This relation is called *Biot and Savart's law*.

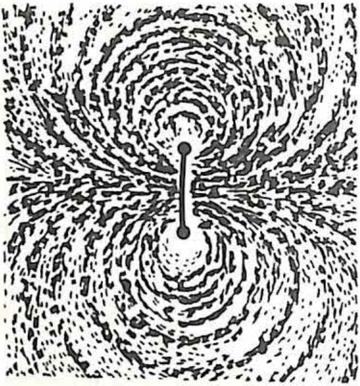


Fig. 54. Force lines of the magnetic field of a circular current detected by the action of the field on iron filings

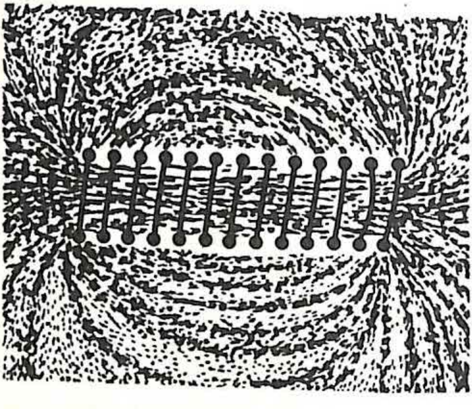


Fig. 55. Force lines of the magnetic field of a solenoid detected by the action of the field on iron filings

The magnetic field intensity of a long straight conductor with current (this and the following formulas are given in both the CGSM and SI systems) is

$$H=\frac{2i}{a} \quad \text{or} \quad H=\frac{i}{2\pi a} \qquad (4.56)$$

where a is the distance from the conductor to the point of the field at which the intensity is being determined.

The magnetic field intensity at the centre of a circular current is

$$H=\frac{2\pi i}{R} \quad \text{or} \quad H=\frac{i}{2R} \qquad (4.57)$$

where R is the radius of the loop.

The magnetic field intensity inside a toroid (Fig. 57) is

$$H=\frac{2Ni}{r} \quad \text{or} \quad H=\frac{Ni}{2\pi r} \qquad (4.58)$$

where N is the total number of turns of the conductor, and r—the average radius of the toroid.

The field intensity inside a straight solenoid whose length considerably exceeds the diameter of its turns is

$$H = 4\pi ni \quad \text{or} \quad H = ni \qquad (4.59)$$

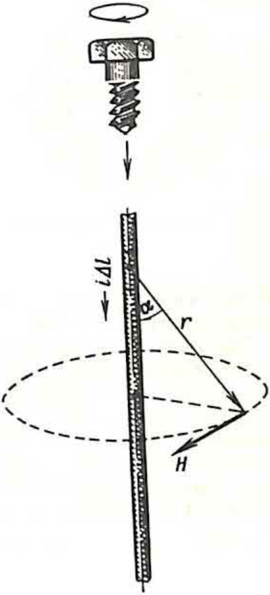


Fig. 56. Illustration of Biot and Savart's law. Right-hand screw rule

where n is the number of turns per unit of length of the solenoid. The magnetic field intensity in such a solenoid has the same magnitude and direction at all points, i.e. the field is *homogeneous*.

The magnetic field intensity of a moving charged particle (Fig. 58) is

$$H=\frac{qv\sin\theta}{r^2} \quad \text{or} \quad H=\frac{qv\sin\theta}{4\pi r^2} \qquad (4.60)$$

where v is the velocity of the particle, r—the distance from the particle to the point of the field at which the intensity is being determined, and θ—the angle between the direction of the velocity and a line drawn from the particle to the given point of the field.

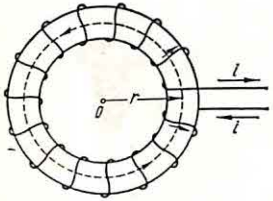


Fig. 57. Toroid

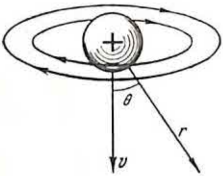


Fig. 58. Magnetic field of a moving particle

The unit of magnetic field intensity in the CGSM system is the oersted (Oe), and in the SI system—the ampere per metre (A/m). 1 A/m is the intensity of a magnetic field induced by an infinitely long straight conductor with a current of 4π A at a distance of two metres from it. 1 Oe is the intensity of a magnetic field induced by an infinitely long straight conductor with a current of a CGSM unit (10 A) at a distance of 2 cm from it;

$$1\ \text{A/m} = 4\pi \times 10^{-3}\ \text{Oe}$$

4. Work Performed when a Conductor with Current Moves in a Magnetic Field. Electromagnetic Induction

When a conductor with current moves in a magnetic field, work is performed:

$$A = i\,(\Phi_2 - \Phi_1) \qquad (4.61)$$

where Φ_1 is the magnetic flux through the current loop at the beginning of motion, and Φ_2—the magnetic flux at the end of motion.

The *magnetic flux* through a loop (in a homogeneous field) is the product of the magnetic induction B, the area of the loop S, and the cosine of the angle α between the direction of the field and a perpendicular to the surface of the loop (Fig. 59):

$$\Phi = BS \cos \alpha \qquad (4.62)$$

The unit of magnetic flux in the CGSM system is the maxwell (Mx), and in the SI system—the weber (Wb).

A changing magnetic flux induces an electric field with closed force lines (a vortex electric field). The induced field manifests itself in the conductor as the action of extraneous forces (p. 136). This phenomenon is called *electromagnetic induction*, and the e.m.f. that appears—the *induced e.m.f.*

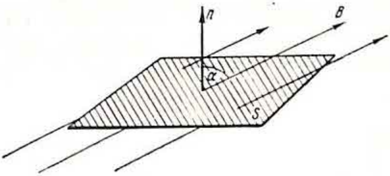


Fig. 59. Magnetic flux through surface S

The currents due to the induced e.m.f. are called *induced currents*.

An induced current has such a direction that its magnetic field prevents a change in the magnetic field which caused the induced current to appear (*the Lenz law*).

The magnitude of the induced e.m.f. can be calculated by the formula

$$\mathscr{E} = -\frac{\Delta\Phi}{\Delta t} \qquad (4.63)$$

Thus, the induced e.m.f. is equal in absolute value to the rate of change of the magnetic flux through the area confined by a loop.

The signs of the e.m.f. and $\frac{\Delta\Phi}{\Delta t}$ are opposite (in accordance with the Lenz law).

5. Self-Induction

Any change in the current in a conductor leads to the appearance of an induced e.m.f. due to the magnetic flux of the current. This phenomenon is called *self-induction*.

The self-induced e.m.f. can be computed by the formula:

$$\mathscr{E} = -L\frac{\Delta i}{\Delta t} \qquad (4.64)$$

where L is the *self-inductance*, and $\frac{\Delta i}{\Delta t}$ — the rate of change of the current. L depends on the geometrical shape and dimensions of the conductor and on the properties of the medium.

The unit of inductance in the SI system is the henry (H), and in the CGSM system — the centimetre (cm). A henry is the inductance of a conductor in which a change of current of 1 ampere per second induces an e.m.f. of 1 volt;

$$1\ \text{H} = 10^9\ \text{cm}$$

All the following formulas are given in the CGSM and SI systems. The inductance of a solenoid with a core is

$$L = k\frac{4\pi\mu S N^2}{l} \quad \text{or} \quad L = k\frac{\mu\mu_0 S N^2}{l} \qquad (4.65)$$

where μ is the permeability, N — the number of turns, S — the cross-sectional area of the solenoid, l — the length of the wound-on wire, and k — a coefficient depending on the ratio of the length of the wound-on wire to the diameter of the coil $\frac{l}{d}$. The values of k are given in Table 107. It should be noted that when calculating L by formula (4.65) one must remember that the value of μ for ferromagnetic materials depends on the shape of the core.

The inductance of a coaxial cable of length l is

$$L = 2\mu l \ln\frac{R_2}{R_1} \quad \text{or} \quad L = \frac{\mu\mu_0 l}{2\pi}\ln\frac{R_2}{R_1} \qquad (4.66)$$

where R_2 and R_1 are the radii of the external and internal cylinders.

The inductance of a two-conductor line of length l and radius of the conductors r (for $r \ll a$) is

$$L = 4\mu l \ln\frac{a}{r} \quad \text{or} \quad L = \frac{\mu\mu_0 l}{\pi}\ln\frac{a}{r} \qquad (4.67)$$

where a is the distance between the axes of the conductors.

The energy of the magnetic field around a conductor with a current is

$$W = \frac{1}{2} Li^2 \tag{4.68}$$

Energy is distributed in a space where there is a magnetic field. The energy density of a homogeneous magnetic field (the energy per unit volume) is determined by the formulas

$$w = \frac{\mu H^2}{8\pi} \quad \text{or} \quad w = \frac{\mu\mu_0 H^2}{2} \tag{4.69}$$

The lifting force of an electromagnet is

$$F = \frac{B^2 S}{8\pi} \quad \text{or} \quad F = \frac{B^2 S}{2\mu_0} \tag{4.70}$$

where S is the cross-sectional area of the shoe of the electromagnet.

Eddy currents (or *Foucault currents*) are induced currents appearing in massive conductors placed in a varying magnetic field.

6. Magnetic Properties of a Substance

A magnetic moment appears in all bodies placed in a magnetic field. This phenomenon is called *magnetization*.

A magnetized body (a magnetic) creates an additional magnetic field with the induction B' which interacts with the induction $B_0 = \mu_a H$ due to macroscopic currents. Both fields produce a resulting field with the induction B which is obtained by vector summation of B' and B_0.

Currents in the molecules of a substance circulate along closed circuits; each of these currents has a magnetic moment (p. 164); in the absence of an external magnetic field the molecular currents are oriented chaotically and the mean current which they create equals zero. Under the action of a magnetic field the magnetic moments of the molecules become oriented mainly along the field, and as a result the substance becomes magnetized. The measure of magnetization of a substance (a magnetic) is the magnetization vector. The *magnetization vector* I equals the vector sum of all the magnetic moments p_m of the molecules confined in a unit volume of the substance:

$$I = \frac{1}{V} \sum p_m$$

The magnetization vector is proportional to the magnetic field intensity vector:

$$I = \varkappa H \tag{4.71}$$

The quantity $\varkappa$ is called the *magnetic susceptibility*. It is a dimensionless quantity.

In the SI system	In the CGSM system	
$B' = \mu I$	$B' = 4\pi I$	(4.72)
$B = \mu_0 H + \mu_0 I$	$B = H + 4\pi I$	(4.73)
$\mu = 1 + \varkappa$	$\mu = 1 + 4\pi\varkappa$	(4.74)

A curve expressing the relation between H and B or H and I is called a *magnetization curve*.

Substances for which $\varkappa > 0$ (but only slightly greater) are called *paramagnetic substances* (or *paramagnetics*); those for which $\varkappa < 0$ are called *diamagnetic substances* (or *diamagnetics*). Substances for which $\varkappa$ is much greater than unity are called *ferromagnetics*.

Ferromagnetics differ from paramagnetics and diamagnetics in a number of properties.

(a) The magnetization curve of ferromagnetics has an intricate nature (see Fig. 60). For paramagnetics it is a straight line with a positive angular coefficient, and for diamagnetics it is a straight line with a negative angular coefficient. The magnetic susceptibility and permeability of ferromagnetics depends on the field intensity; such a relation is absent for paramagnetics and diamagnetics.

For ferromagnetics the initial permeability (μ_{in}) is usually indicated. It is the limiting value of the permeability when the field intensity and induction are close to zero, i.e.

$$\mu_{in} = \lim_{H \to 0} \mu$$

The curve of μ versus H for ferromagnetics passes through a maximum (see Fig. 62*a*). The maximum value μ_{max} is also usually given in the relevant tables.

(b) The magnetic susceptibility of ferromagnetics grows with increasing temperature. At a certain temperature T_C a ferromagnetic transforms into a paramagnetic; this temperature is called the *Curie point*. At a temperature above the Curie point a substance is a paramagnetic. Near the Curie point the magnetic susceptibility of a ferromagnetic sharply grows.

The magnetic susceptibility of diamagnetics and certain paramagnetics (for example, in the alkali metals) does not depend on the temperature. The magnetic susceptibility of paramagnetics (with a few exceptions) changes in inverse proportion to the absolute temperature.

(c) A demagnetized ferromagnetic is magnetized by a magnetic field; the dependence of B (or I) on H upon magnetization will be expressed by curve *0-1* (Fig. 60). This curve is called the *initial magnetization curve*. The magnetization in weak fields grows rapidly, then the growth decelerates and, finally, a state of saturation sets in when the magnetization remains virtually constant upon a further increase in the field.

The maximum value of the magnetization is called the *saturation magnetization* (I_{sat}).

When H drops to zero, B (and I) will change along curve *1-2*; the change in induction will lag behind the change in field intensity. This phenomenon is called *magnetic hysteresis.*

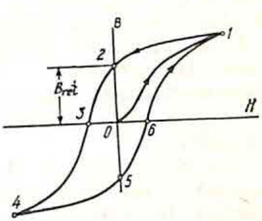


Fig. 60. Hysteresis loop: *0-1*—curve of magnetization from unmagnetized state; *1-2-3*—demagnetization curve

The value of the induction remaining in a ferromagnetic after the field is removed (when $H = 0$) is called the *magnetic retentivity* (or simply the *retentivity*) B_{ret}. In Fig. 60 the value of B_{ret} equals the segment *0-2*. To demagnetize a ferromagnetic, the retentivity must be removed. To do this a field of the opposite direction must be created. The change in the induction in a field of the opposite direction is shown by curve *2-3-4*.

The field intensity H_{coer} (segment *0-3* in Fig. 60) at which the induction equals zero is called the *coercive intensity (force)*.

The dependence of B (or I) on the periodically changing magnetic field intensity from $+H$ to $-H$ is expressed by closed curve *1-2-3-4-5-6-1*. Such a curve is called a *hysteresis loop*.

During one cycle of change in the field intensity from $+H$ to $-H$ energy proportional to the area of the hysteresis loop will be consumed.

The properties of ferromagnetics are explained by the presence in them of regions which in the absence of an external magnetic field are spontaneously magnetized to saturation. These regions are called *domains*. But the arrangement and magnetization of these regions are such that in the absence of a field the total magnetization of the entire body equals zero.

When a ferromagnetic is in a magnetic field, the boundaries between the domains become displaced (in weak fields) and the magnetization vectors of the domains turn in the direction of the magnetizing field (in stronger fields), as a result of which the ferromagnetic becomes magnetized.

A ferromagnetic placed in a magnetic field changes its linear dimensions, i.e. deforms. This phenomenon is called *magnetostriction*. The relative elongation depends on the nature of the ferromagnetic and the magnetic field intensity.

The magnitude of the magnetostriction effect does not depend on the direction of the field; in some substances contraction is observed (nickel), in others elongation (iron in weak fields) along the field. This phenomenon is used to obtain ultrasonic oscillations with frequencies up to 100 kHz.

TABLES AND GRAPHS

Magnetic Field of the Earth

The Earth is surrounded by a magnetic field. The points of the Earth at which the magnetic field intensity is directed vertically are called its *magnetic poles*. The Earth has two such poles: the north magnetic pole (in the southern hemisphere) and the south magnetic pole (in the northern hemisphere).

The straight line passing through the magnetic poles is called the Earth's *magnetic axis*. The circumference of the great circle in the plane perpendicular to the magnetic axis is called the *magnetic equator*. The magnetic field intensity at points on the magnetic equator is directed horizontally.

The magnetic field intensity at the magnetic equator is about 0.34 Oe, and at the magnetic poles about 0.66 Oe. In some regions (in so-called regions of magnetic anomalies) the intensity sharply grows. In the region of the Kursk magnetic anomaly it reaches 2 Oe.

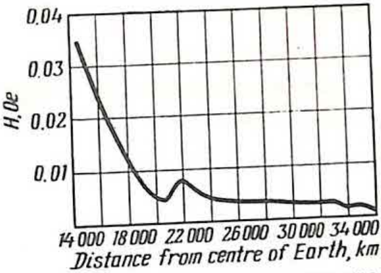


Fig. 61. Intensity of the Earth's magnetic field at high altitudes

Table 98

Properties of Steels Used in Electrical Engineering

Grade of steel	μ_{in}, Gs/Oe	μ_{max}, Gs/Oe	H_{coer}, Oe	B (at 25 Oe), Gs	ρ, ohm× ×mm²/m
Э31	250	5 500	0.55	14 600	0.5
Э41	300	6 000	0.45	14 600	0.6
Э42	400	7 500	0.4	14 500	0.6
Э45	600	10 000	0.25	14 600	0.6
Э310	1000	30 000	0.12	17 500	0.5

Note. For the symbols used see pp. 172-173.

Table 99

Properties of Selected Iron-Nickel Alloys

Alloy	μ_{in}, Gs/Oe	μ_{max}, Gs/Oe	H_{coer}, Oe	I_{sat}, Gs	ρ, ohm× ×mm²/m
70HM	20 000	100 000	0.03	8 000	0.55
80HXC	35 000	120 000	0.015	7 000	0.62
50HCX	3 000	30 000	0.2	10 000	0.85
50H	3 000	35 000	0.12	15 000	0.45
65HП	3 000	100 000	0.1	13 000	0.35
50HП	2 000	20 000	0.2	15 500	0.45
Mo-permalloy	20 000	75 000	0.03	8 500	0.55
78.5 Ni-permalloy	10 000	100 000	0.025	10 700	0.16

Note. These alloys have a high permeability, which drops sharply at high field intensities and high frequencies. In addition, the value of μ greatly depends on the mechanical stresses.

Table 100

Properties of Selected Magnetically Refractive Materials

Material of magnet	H_{coer}, Oe	B_{ret}, Gs	$\frac{HB}{8\pi}$, erg/cm³
Alloys: alni 1 (AH1)	250	7 000	28 000
alni 3 (AH3)	500	5 000	36 000
alnico 12 (AHKO 1) . .	500	6 800	55 000
alnico 18 (AHKO 3) . .	650	9 000	97 000
alnisi (AHK)	750	4 000	43 000
magnico (AHKO 4) . .	500	12 300	150 000
Barium ferrites	1 600-2 900	1 800-4 000	30 000-150 000
Platinum magnetic alloys	1 500-4 000	3 000-6 000	10 000-15 000
Steels: EX3	60	9 500	12 000
E7B6	62	10 000	13 000
EX5K5	100	8 500	18 000
EX9K15M . .	170	8 000	28 000

Note. These materials are characterized by a high coercive force and are used to manufacture permanent magnets. An important characteristic of these materials is the maximum value of the quantity $\frac{HB}{8\pi}$. This quantity is proportional to the maximum energy of the magnetic field surrounding the ferromagnetic.

Table 101

Properties of Magneto-Dielectrics

Material	μ, Gs/Oe	α, 10^{-6} K^{-1}
Pressperm T4-180	160-200	+400
Alsifer T4-90	75-85	+400
Alsifer T4-60	55-65	−300, −400
Alsifer B4-32	30-34	−200, +250
Carbonyl iron K-12	11-14	−50, +50
Ferroelast K-9	9-10	−50, +50
Alsifer P4-6	5-8	−80, −150

Note. Magneto-dielectrics consist of fine ferromagnetic particles (10^{-1} to 10^{-4} cm) which are bound together with a dielectric. The resistivity of these materials ranges from 1 to 400 ohm·cm; α is the temperature coefficient of resistivity.

Table 102

Principal Properties of Ferrites

Ferrite	μ_{in}, Gs/Oe	α, 10^{-6} K^{-1}	ρ, ohm·cm
Manganese-zinc ferrites:			
4000HM	4000	2	10^2
3000HM	3000	3	
2000HM	2000	0.6-1.5	
1500HM	1500	0.6-1.5	
1000HM	1000	1.5	
Nickel-zinc and lithium-zinc ferrites:			
2000HH	2000	3	10^4-10^7
600HH	600	6	
400HH	400	5	
200HH	200	4-25	
100HH	100	10-30	
50BЧ	50	50	

Note. Ferrites are mixtures of metal oxides (of nickel, zinc, and iron) subjected to special heat treatment as a result of which they acquire a high resistivity; α is the temperature coefficient of resistivity.

Table 103

Permeability of Paramagnetics and Diamagnetics

Paramagnetics	$(\mu-1)$, 10^{-6}	Diamagnetics	$(1-\mu)$, 10^{-6}
Nitrogen . . .	0.013	Hydrogen . .	0.063
Air	0.38	Benzene . . .	7.5
Oxygen	1.9	Water	9.0
Ebonite . . .	14	Copper	10.3
Aluminium .	23	Glass	12.6
Tungsten . . .	176	Rock salt . .	12.6
Platinum . .	360	Quartz	15.1
Liquid oxygen	3400	Bismuth . . .	176

Table 104

Curie Point of Metals

Substance	T_C, °C	Substance	T_C, °C
Gadolinium	20	Magnetite	585
Permalloy, 30%	70	Iron, electrolytic	769
Heusler alloy	200	Iron, resmelted in hydrogen	774
Nickel	358	Cobalt	1140
Permalloy, 78%	550		

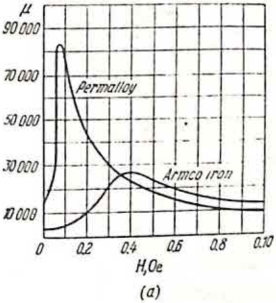
Table 105

Unit Susceptibility of Selected Metals (per gramme at 18 °C in CGSM System)

Metal	$\varkappa_\rho$, 10^{-6}	Metal	$\varkappa_\rho$, 10^{-6}
Aluminium	0.58	Manganese	7.5
Antimony	−0.87	Mercury	−0.19
Cadmium	−0.18	Selenium	−0.32
Calcium	0.5	Silver	−0.20
Chromium	3.6	Sodium	0.6
Copper	−0.086	Tellurium	−0.31
Germanium	−0.12	Tin	0.03
Indium	−0.11	Tungsten	0.28
Lead	−0.12	Vanadium	1.4
Lithium	0.5	Zinc	−0.157

Note. The unit susceptibility $\varkappa_\rho$ equals the ratio of the susceptibility $\varkappa$ to the density of the substance ρ: $\varkappa_\rho = \frac{\varkappa}{\rho}$.

Dependence of Permeability and Induction on the Magnetic Field Intensity (in Primary Magnetization)



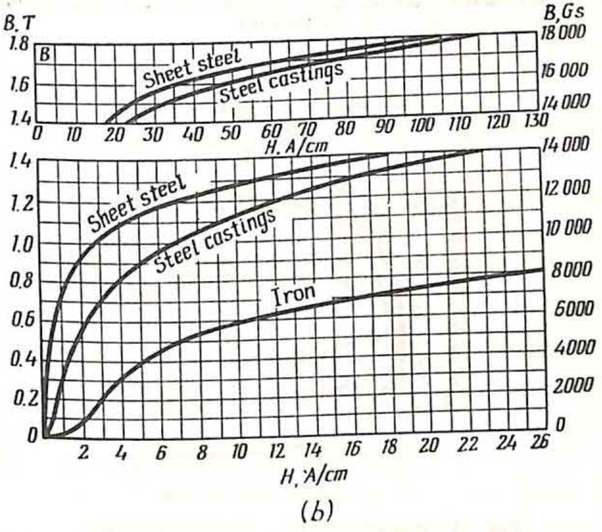


Fig. 62. (*a*) Dependence of the permeability of iron and Permalloy on the intensity in weak fields; (*b*) dependence of the induction in steel and iron on the magnetic field intensity

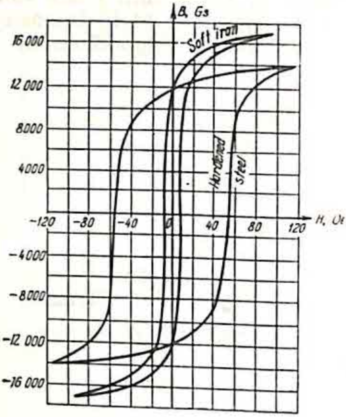


Fig. 63. Hysteresis loop for soft iron and tempered steel (about 1% of carbon)

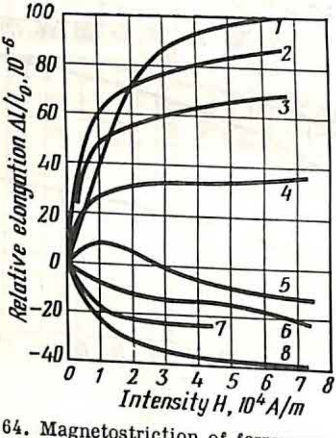


Fig. 64. Magnetostriction of ferromagnetics:
1—54% Pt, 46% Fe; *2*—70% Co, 30% Fe; *3*—50% Co, 50% Fe; *4*—50% Ni, 50% Fe; *5*—iron; *6*—cobalt (annealed); *7*—ferrite 20% Ni, 80% Zn; *8*—nickel

Table 106

Induction and Hysteresis Losses in Ferromagnetics and Ferrites

Material	Induction B (Gs) at H (Oe) equal to						Hysteresis losses, erg/cm^3 × cycle
	0.1	0.5	2.0	10.0	50	500	
Ferrite Mg-Mn	—	100	2 000	2 300	—	—	—
Ditto, Mn-Zn	80	500	2 300	3 600	—	—	—
Ditto, 70% Ni-Cu	—	—	—	600	1 000	—	—
Ditto, 30% Ni-Fe	—	—	—	2 500	3 100	—	—
Ditto, Ni-Zn	5	80	130	1 500	2 400	—	—
Iron, annealed	100	750	14 000	16 500	17 200	21 000	600
Ditto, cast, annealed . .	—	—	600	5 000	8 500	14 000	10 000
Ditto, cobalt (35% Co) . .	—	—	4 000	15 000	21 000	24 200	3 500
Ditto, electrolytic	40	500	11 000	15 000	17 000	21 000	2 500
Ditto, silicon, sheet (4.3% Si) . .	200	4 500	10 000	13 500	15 300	19 500	690
Steel, mild (0.1% C) . .	30	300	6 000	14 000	17 000	21 000	5 000
Ditto, sheet .	40	400	9 000	14 500	16 500	21 000	2 500

Notes. 1. The numerical values should be considered as approximate because they may vary for different samples.

2. The values of the losses per remagnetizing cycle in 1 cm^3 of a substance are indicated for a hysteresis loop with the maximum value of the induction of 1000 Gs.

Table 107

Values of Coefficient k for Calculating Inductance

Ratio of length of winding to diameter $\left(\frac{l}{d}\right)$	0.1	0.5	1	5	10
k	0.2	0.5	0.6	0.9	appr. 1.0

Note. For $\frac{l}{d} \geqslant 10$ the value of k is approximately unity.

D. ALTERNATING CURRENT

FUNDAMENTAL CONCEPTS AND LAWS

An *alternating current* is such a current whose intensity or direction (or both simultaneously) changes with time.

Currents that change only in magnitude are called *pulsating currents.*

Alternating currents that change sinusoidally (Fig. 65) are used most frequently. Periodic non-sinusoidal currents can with any degree of accuracy be represented as the sum of sinusoidal alternating currents (see p. 103).

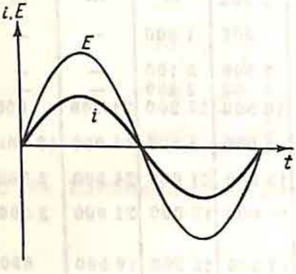


Fig. 65. Graphs of change in alternating e.m.f. and current (sine law, $\varphi = 0$)

The instantaneous values of a sinusoidal alternating current and voltage are given by the formulas

$$i = I_0 \sin \omega t \quad (4.75)$$

$$u = U_0 \sin (\omega t + \varphi) \quad (4.76)$$

where I_0 and U_0 are the maximum (amplitude) values of the current and voltage, ω—the angular (cyclic) frequency of the current, t—the time, φ—the phase shift between the current and voltage (see p. 102), $\omega = 2\pi f$, and f—the frequency of the current.

The *effective value of an alternating current* (I) is the value of the direct current which develops the same power as the given alternating current when flowing through the same active (ohmic) resistance.

In most cases (but not always!) ammeters and voltmeters show the effective value of the current or the voltage (U).

For sinusoidal currents

$$I = \frac{I_0}{\sqrt{2}} \quad \text{and} \quad U = \frac{U_0}{\sqrt{2}} \quad (4.77)$$

The mean power developed by an alternating current in a circuit is

$$P = UI \cos \varphi \quad (4.78)$$

The quantity $\cos \varphi$ is called the *power factor.*

An inductance L in an alternating-current circuit functions similar to a resistance in the circuit, i.e. it reduces the current.

The magnitude of the inductive reactance is

$$R_L = \omega L \tag{4.79}$$

This reactance is due to the appearance of a self-induction e.m.f. in the coil.

An alternating current in a circuit containing only an inductive reactance lags in phase behind the voltage applied across the circuit by 90°.

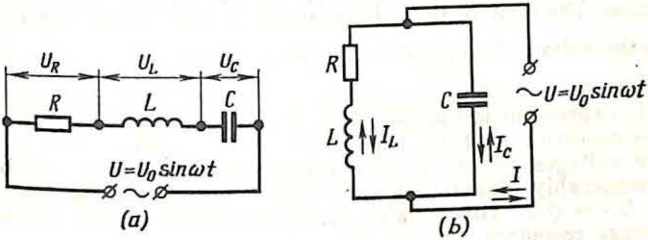


Fig. 66. Series (*a*) and parallel (*b*) resonance circuits

A capacitance in an alternating-current circuit passes the current (in contrast to a direct current!). The resistance of a capacitance to the flow of an alternating current is called *capacitive reactance.*

The capacitive reactance is

$$R_C = \frac{1}{\omega C} \tag{4.80}$$

The current in a capacitor leads the voltage by 90°.

When an ohmic resistance, inductive reactance and capacitive reactance are connected in series (Fig. 66*a*), their total action is

$$Z = \sqrt{R^2 + (R_L - R_C)^2} \tag{4.81}$$

The quantity Z is called the *impedance*, in contrast to the *ohmic resistance* R.

The amplitude of the current in a series resonance circuit is

$$I = \frac{U_0}{Z} = \frac{I_0}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}} \tag{4.82}$$

where Q is the quality (see below); the phase difference between the current and voltage is determined by the relations

$$\tan\varphi = \frac{R_L - R_C}{R} \quad \text{and} \quad \cos\varphi = \frac{R}{Z} \qquad (4.83)$$

When $R_L = R_C$ we have $\varphi = 0$, the impedance Z is minimum (see Fig. 68), and the current in the circuit is maximum (see Fig. 69). This phenomenon is called *series electrical resonance.*

In series resonance the voltages across the inductance and capacitance equal each other in value, but are opposite in phase. The ratio of the voltage across a capacitor U_C (or U_L) to the voltage U applied to the circuit equals $\frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = Q$. This quantity is called the *quality of the circuit*; ω_0 in this expression is the resonance frequency, determined from the condition that $R_L = R_C$. At circuit qualities of $Q \gg 1$ the voltages across the inductance and capacitance may be considerably higher than the applied voltage U, i.e. $U_L = U_C = QU$. This is why this phenomenon is also called *voltage resonance.*

When a capacitance and an inductance are connected parallel to an ohmic resistance (Fig. 66*b*), the impedance is

$$Z = R_C \sqrt{\frac{R^2 - R_L^2}{R^2 + (R_L - R_C)^2}} \qquad (4.84)$$

and the phase difference is determined from the relation

$$\tan\varphi = \frac{R_L}{R}\left(1 - \frac{R_L}{R_C}\right) - \frac{R}{R_C} \qquad (4.85)$$

When $\varphi = 0$ we have $R_L \approx R_C$, and the impedance (see Fig. 70) is maximum. This phenomenon is called *parallel electrical resonance.*

In parallel resonance the current I in the common circuit is minimum and coincides in phase with the applied voltage U, while the currents I_L and I_C flowing through the inductance and capacitance are equal in value, but opposite in phase, and the currents in the circuit branches may be considerably greater than the current in the common circuit (at $Q \gg 1$): $I_C = I_L = QI$. This is why parallel resonance is also called *current resonance.*

The impedance Z (at $Q > 1$) has the maximum value Z_{max} in parallel resonance. Graphs showing how $\frac{Z}{Z_{max}}$ depends on the relative frequency $\frac{\omega}{\omega_0}$ are shown in Fig. 70.

When an alternating current flows through a conductor, it generates induced currents; as a result the current density will be greater at the surface of the conductor than in the middle. The difference will be the greater, the higher is the frequency. At high frequencies the current in the middle of the conductor may virtually vanish. This phenomenon is called the *surface effect* (or *skin effect*).

TABLES AND GRAPHS

Resistance to Direct and Alternating Currents

The ratio of the resistances to alternating and direct currents depends on the parameter ξ:

$$\xi = 0.14d\sqrt{\frac{\mu f}{\rho}}$$

where d is the diameter of the wire (cm), f—the frequency (Hz), ρ—the resistivity (ohm·cm), and μ—the permeability.

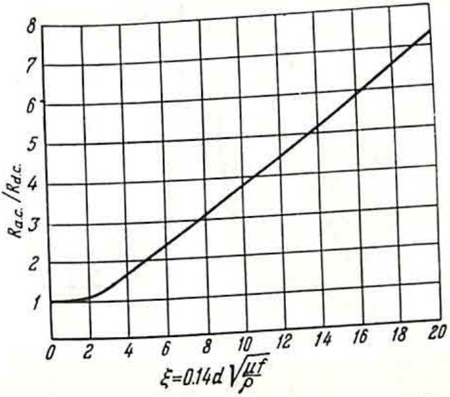


Fig. 67. Graph of dependence of ratio between resistances to alternating and direct currents on ξ

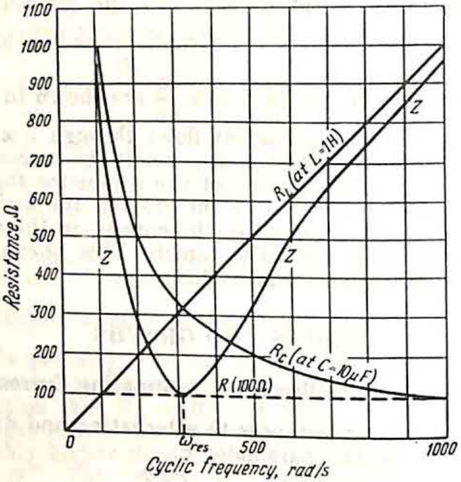


Fig. 68. Change in inductive reactance, capacitive reactance, and impedance depending on frequency in a series resonance circuit

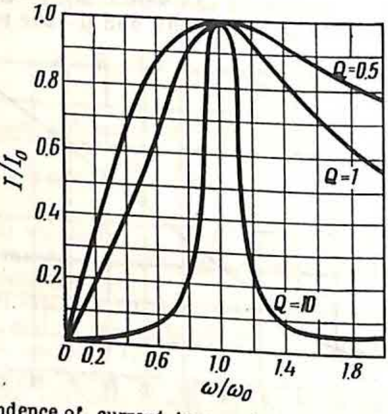


Fig. 69. Dependence of current in a series resonance circuit on frequency. The curves have been calculated by formula (4.82). The relative values $\frac{I}{I_0}$ and $\frac{\omega}{\omega_0}$ are laid off along the coordinate axes

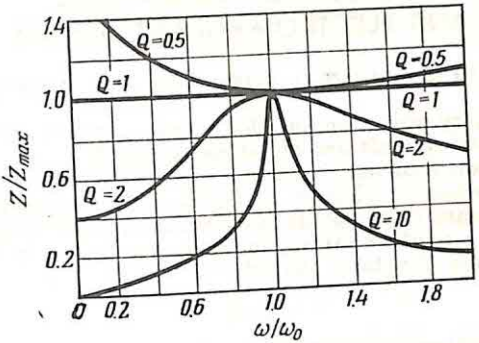


Fig. 70. Dependence of impedance Z on frequency in a parallel resonance circuit. The relative values $\frac{Z}{Z_{max}}$ and $\frac{\omega}{\omega_0}$ are laid off along the coordinate axes. The calculations have been performed for the case when the ohmic resistances in branches L and C are equal

Table 108

Depth of Penetration σ of High-Frequency Currents (for a Straight Round Copper Wire)

Frequency, MHz	0.01	0.1	1	10	100
σ, mm	0.65	0.21	0.065	0.021	0.006

Notes. 1. Calculations for other frequencies and other materials can be performed using the formula

$$\sigma = 503.3\sqrt{\frac{\rho}{\mu f}}$$

where ρ is the resistivity (ohm·mm²/m), μ—permeability of the material, and f—the frequency (Hz).

2. The *depth of penetration* is the distance from the surface of the wire at which the current density is e times less than at the surface; here e is the base of natural logarithms ($e \approx 2.72$).

E. ELECTRIC OSCILLATIONS AND ELECTROMAGNETIC WAVES

FUNDAMENTAL CONCEPTS AND LAWS

Oscillatory variations of the charge, current, or voltage in an electric circuit are called *electric oscillations*. An alternating electric current is an example of electric oscillations.

High-frequency electric oscillations are generated as a rule by means of an oscillating circuit. An *oscillating circuit* is a closed circuit containing inductance L and capacitance C.

The *period of natural oscillations* of a circuit is

$$T = 2\pi \sqrt{LC} \qquad (4.86)$$

This relation is called *Thomson's formula*; it is valid in the absence of energy losses. When there are energy losses in a circuit (for example, when the circuit contains an ohmic resistance R), the natural oscillations of the circuit are damped, and

$$T = \frac{2\pi}{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}} \qquad (4.87)$$

while the current in the circuit changes according to the law of damped oscillations:

$$i = I_0 e^{-\frac{R}{2L}t} \sin \omega t \qquad (4.88)$$

For a graph of damped oscillations see p. 105.

When an alternating e.m.f. is applied to an oscillatory circuit, forced oscillations appear in it. The amplitude of the forced oscillations of the current at constant values of L, C, and R depends on the ratio of the natural frequency of oscillations of the circuit to the frequency of the change in the sinusoidal e.m.f. (see Fig. 69).

According to the Biot-Savart law (see p. 165), a conduction current induces a magnetic field with closed force lines, called a *vortex field*.

An alternating conduction current creates a variable magnetic field. An alternating current in contrast to a direct one passes through a capacitor (see p. 183), but this current is not a conduction one; it is called a *displacement current*. This current is an electric field varying with time; it induces a variable magnetic field, like an alternating conduction current does. The density of a displacement current is

$$j=\frac{\Delta D}{\Delta t} \tag{4.89}$$

where D is the induction vector of the electric field.

At each point of space a change in the induction of the electric field with time induces a variable vortex magnetic field (Fig. 71*a*). The vectors B of the induced magnetic field are in a plane perpendicular to the vector D. The mathematical equation expressing this law is called the *first Maxwell equation*.

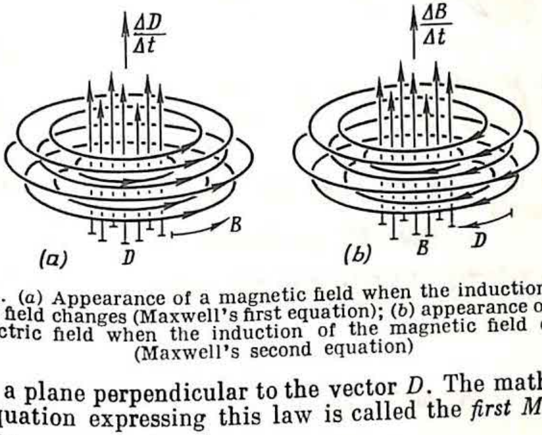


Fig. 71. (*a*) Appearance of a magnetic field when the induction of the electric field changes (Maxwell's first equation); (*b*) appearance of a vortex electric field when the induction of the magnetic field changes (Maxwell's second equation)

In electromagnetic induction an electric field with closed force lines appears (a vortex field) which manifests itself as an induced e.m.f. (see p. 169). At each point of space the change in the magnetic field induction vector with time induces an alternating vortex electric field (Fig. 71*b*). The vectors D of the induced electric field are in a plane perpendicular to the vector B. The mathematical equation expressing this law is called the *second Maxwell equation*.

The complex of varying electric and magnetic fields inseparably linked with one another is called an *electromagnetic field*.

It follows from the Maxwell equations that the change with time of an electric (or magnetic) field appearing at a point will travel from one point to another, and this will be attended by the mutual transformations of the electric and magnetic fields.

Electromagnetic waves are a process of simultaneous propagation of variable electric and magnetic fields in space. The vectors of the electric and magnetic field intensities (E and H) in an electromagnetic wave are perpendicular to each other, while the vector v of the propagation velocity is perpendicular to the plane in which the two vectors E and H are (Fig. 72). This is valid for the propagation of electromagnetic waves in unlimited space.

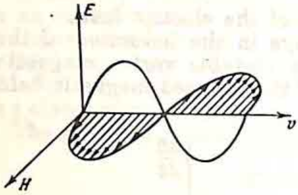


Fig. 72. Mutual arrangement of vectors E, H, and v in an electromagnetic wave

The velocity of propagation of electromagnetic waves in a vacuum does not depend on the wavelength and equals

$$c = (2.997\,925 \pm 0.000\,003) \times 10^{10}\ \text{cm/s}$$

The velocity of electromagnetic waves in different media is less than that in a vacuum:

$$v = \frac{c}{n} \qquad (4.90)$$

where n is the refractive index of the medium (see p. 198).

1. The Electromagnetic Spectrum

The spectrum is plotted to a logarithmic scale (see Table 109).

The first column indicates the wavelengths (at the left in different units of length, at the right in cm). The second column gives the frequencies in Hz, the third column contains the names of the wavelength ranges, the fourth column—the names of the frequencies (or waves). The fifth column indicates the principal ways of generating and using the electromagnetic oscillations.

Low-frequency and radio waves have the lowest frequencies. They are generated by various oscillators.

Table 109

The Electromagnetic Spectrum

Wavelength	Frequency (Hz)		Range	Name of wave (or frequency) group	Principal ways of generation and use
10^8 km	10^{13}	3×10^{-3}	Low-frequency waves	Infralow frequencies	Oscillators of special design
				Low frequencies	
10^6 km 10^5 km	10^{11} 10^{10}	3×10^{-1}		Industrial frequencies	Alternating-current generators (alternators); most electrical appliances and motors are supplied with alternating current at 50 to 60 Hz
10^3 km	10^8	3×10^2		Sonic (audio) frequencies	Audio-frequency oscillators. Used in electroacoustics (microphones, loudspeakers), cinematography, and radio broadcasting
			Radio waves	Long	Electric oscillators of various designs. Used in telegraph, radio broadcasting, television, radar, etc.
1 km	10^5	3×10^5		Medium	
				Short	
				Metre	Used for studying the properties of a substance
1 m 1 dm	10^2 10	3×10^8 3×10^9		Decimetre	

Table 109 (concluded)

Wavelength		Frequency (Hz)	Range	Name of wave (or frequency) group	Principal ways of generation and use
			Radio waves	Centimetre	Generated in magnetrons, klystron oscillators, and masers. Used in radar, microwave spectroscopy, and radio astronomy
1 cm	1	3×10^{10}		Millimetre	
				Intermediate	
1 mm	10^{-1}	3×10^{11}	Infra-red rays	Decamicron	Radiation of heated bodies (arc and gaseous discharge lamps, etc.). Used in infrared spectroscopy, and for photography in the dark (in infrared rays)
				Micron	
			Light rays		
1 µm	10^{-4}	3×10^{14}	Ultraviolet rays	Near	Radiation of Sun, mercury-discharge lamps, etc. Used in ultraviolet microscopy, and in medicine
				Far	
			X-rays	Ultrasoft	Generated in X-ray tubes and other devices where electrons having an energy over 10^3 eV are decelerated. Used in medicine, for studying the structure of a substance, and in flaw detection
1 nm	10^{-7}	3×10^{17}		Soft	
1 Å	10^{-8}	3×10^{18}		Hard	
1 X	10^{-11}	3×10^{21}	Gamma-rays		Appear in radioactive decay of nuclei, in the deceleration of electrons having an energy over 10^5 eV, and in other interactions of elementary particles. Used in gamma flaw detection, and in studying the properties of a substance

Infrared rays are emitted mainly upon the oscillations of molecules or groups of atoms.

Light waves are emitted by atoms and molecules of a substance as a result of a change in the states of the electrons on their outer shells (see p. 230).

Ultraviolet rays have the same origin as light waves.

X-rays are emitted as a result of changes in the states of electrons on the inner shells of atoms (*characteristic radiation*) or as a result of sharp deceleration of electrons and other charged particles.

Gamma-rays are emitted by excited nuclei of atoms and by elementary particles as a result of various interactions.

Further information on certain properties of various kinds of waves is given in Chapter 5.

2. Radiation of Electromagnetic Waves

Charged particles travelling with acceleration emit electromagnetic waves.

A dipole (see p. 129), the distance between whose charges changes according to the law $l = l_0 \sin \omega t$ (provided that $l_0 < \lambda$, where λ is the length of an electromagnetic wave) emits during one second the mean energy

$$u = 1.11 \times 10^{-16} q^2 \omega^4 l_0^2 \text{ W}$$

where q is the charge of a dipole (in C).

Any conductor along which alternating currents flow is an emitter of electromagnetic waves. The most effective radiation occurs when the dimensions of the emitter are compatible with the length of the emitted wave.

Conductors effectively emitting (or receiving) electromagnetic waves are called *antennas* or *aerials*.

The density of the energy flux (the mean energy carried by electromagnetic waves in one second through an area of one square metre at right angles to the direction of their propagation) is

$$S = EH \text{ W} \tag{4.91}$$

A current element $i\Delta l$ changing according to the harmonic law $i = I_0 \sin \omega t$ generates at a distance r from it that is much greater than the emitted wavelength λ and the length of the element Δl an electromagnetic field with an electric field intensity E_θ and a magnetic field intensity H_θ:

$$E_\theta = -\frac{1}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\, I_0 \frac{\Delta l}{\lambda r} \sin\theta \cdot \sin(\omega t - kr) =$$

$$= -188.27 I_0 \frac{\Delta l}{\lambda r} \sin\theta \cdot \sin(\omega t - kr) \text{ V/m} \qquad (4.92)$$

$$H_\theta = -\frac{1}{2} I_0 \frac{\Delta l}{\lambda r} \sin\theta \cdot \sin(\omega t - kr) \text{ A/m} \qquad (4.93)$$

In these expressions θ is the angle between the straight line connecting the current element $i\Delta l$ to the point of observation and the direction of the conductor, and $k = \frac{2\pi}{\lambda}$ is the wave number.

The total mean energy emitted by a current element is

$$\widetilde{W} = 788.6 \left(\frac{i\Delta l}{\lambda}\right)^2 \text{ W} \qquad (4.94)$$

CHAPTER V

OPTICS

FUNDAMENTAL CONCEPTS AND LAWS

Light is electromagnetic radiation with a wavelength from 0.4 to 0.8 μm. A source of such waves are atoms and molecules in which a change in the energy state of the electrons occurs (see p. 228).

1. Photometry

Photometry studies the intensity of light, brightness, and illumination both according to the visual perception by the eye and according to the radiant flux.

The energy emitted by a body in one second is called the *radiated power*. The energy transmitted by a light wave through an area in one second is called the *radiation flux* through this area. The radiation flux estimated by visual perception by the eye is called the *luminous flux*. Since the sensitivity of the eye varies for different wavelengths, the ratio between the radiation flux and the luminous flux is different for different wavelengths. Usually in daytime vision the eye is most sensitive to light having a wavelength of 5550 Å. The ratio of the radiated power at a wavelength of 5550 Å to the radiated power at a wavelength of λ causing the same visual sensation is called the *relative sensitivity of the eye*, or the *relative luminosity* (K_λ). The graphical dependence of K_λ on λ is called the *curve of relative luminosity*.

In twilight vision the eye has the maximum sensitivity to light with wavelengths of about 5070 Å.

In daylight vision one watt of radiant energy with a wavelength of 5550 Å corresponds to a luminous flux of 680 lumens (see below). In twilight vision one watt of radiant energy with a wavelength of 5070 Å corresponds to a luminous flux of 1745 lumens.

The *luminous intensity* (I) is the luminous flux emitted by a point source of light into a unit solid angle:

$$I=\frac{\Delta\Phi}{\Delta\Omega} \tag{5.1}$$

where $\Delta\Phi$ is the luminous flux, and $\Delta\Omega$—a small solid angle.

The candela (cd) has been adopted as the unit of luminous intensity.

A standard of a special design is used to determine the luminous intensity equal to one candela. Its radiation corresponds to the radiation of a black body (see p. 213) at the solidification point of platinum (2042 K). The luminous intensity emitted by such a source in the direction of a normal to its surface from an area of $\frac{1}{60}$ cm^2 equals one candela.

The unit of luminous flux is the lumen (lm). One lumen equals the luminous flux contained in a solid angle of one steradian at a luminous intensity of one candela.

The *illumination* (E) is the luminous flux per unit of area:

$$E=\frac{\Delta\Phi}{\Delta S} \tag{5.2}$$

where ΔS is the area of the surface receiving the luminous flux.

The units of illumination are the lux (lx) and the phot (ph):

$$1 \text{ lx} = 1 \text{ lm/m}^2 \text{ and } 1 \text{ ph} = 1 \text{ lm/cm}^2$$

The illumination E of a flat surface created by a point source is

$$E=\frac{I\cos\alpha}{r^2}$$

where I is the luminous intensity, r—the distance to the illuminated surface, and α—the angle between the direction of the luminous flux and a normal to the surface.

The *luminance* (B) is a measure of the radiation of a luminescent surface:

$$B = \frac{I}{S} \tag{5.3}$$

where S is the area of the visible surface (the projection of the observed surface onto a plane perpendicular to the direction of observation), and I—the luminous intensity.

The units of luminance are the nit (nt) and the stilb (sb).

One nit is the luminance of a source which emits light with an intensity of one candela from 1 m^2 of luminous surface;

$$1 \text{ sb} = 10^4 \text{ nt}$$

The luminance of the standard used to determine luminous intensity equals 60 sb.

The *intensity of light* is the average flux of electromagnetic waves through a unit of surface at right angles to the direction of propagation of the waves in time. The intensity of light is proportional to the square of the amplitude of E (or H) (see p. 193).

2. Principal Laws of Geometrical Optics

It is assumed in geometrical optics that in a homogeneous medium light travels in a straight line.

The *angle of incidence* is the angle between the incident ray and the normal to the interface between two media at the point of incidence. The *angle of reflection* is the angle between this normal and the reflected ray. The *angle of refraction* is the angle between the normal and the refracted ray.

When a ray is incident on the interface between two media the angle of incidence is equal to the angle of reflection. The incident ray, the normal, and the reflected ray all lie in the same plane.

The magnitude of reflection is characterized by the *reflection coefficient* ρ, which is equal to the ratio of the energy flux in the reflected wave to that in the incident wave.

(The reflection coefficient is frequently expressed in per cent.)

Two kinds of light reflection are distinguished depending on the properties of the interface.

If the interface has irregularities whose dimensions are considerably smaller than the wavelength of light, then *regular* (or *mirror*) *reflection* occurs. In this case the incident parallel rays of light after reflection remain parallel.

If the irregularities are arranged chaotically on the reflecting surface and have dimensions compatible with the wavelength of light, *diffuse reflection* occurs. In this case the parallel rays of light after reflection stop being parallel; however, if $h \cos i \ll \lambda$ (h is the size of the irregularities, and i—the angle of incidence of the rays), mirror reflection will occur.

The ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for a given wavelength. The incident and refracted rays, and the normal to the plane at the point of incidence are in one plane:

$$\frac{\sin i}{\sin r}=n \tag{5.4}$$

The quantity n is called the *relative index of refraction* of the second medium; it equals the ratio of the velocities of light in the two media:

$$n=\frac{v_1}{v_2}$$

The index of refraction with respect to a vacuum is called the *absolute index of refraction* of the medium. The value of n depends on the wavelength.

A ray passing from a medium with a greater index of refraction to one with a smaller index may be reflected completely (*total reflection*). The angle of incidence i_{cr} beginning from which the entire quantity of light is reflected from the interface is called the *critical angle of total reflection*. The magnitude of the angle i_{cr} is found from the relation

$$\sin i_{\mathrm{cr}}=\frac{1}{n}$$

where n is the index of refraction of the medium in which total reflection occurs relative to the medium in which the light propagates.

The rays of light passing through a prism are refracted (Fig. 73). The angle P of the prism is called the *refracting* or *prism angle*. The continuations of the entering and emerging rays form the angle δ called the *deviation angle*:

$$\delta = i_1 + i_2 - P, \qquad P = r_1 + r_2$$

The angle δ is the smallest (for a given value of the angle P) when $i_1 = i_2$ (and, therefore, $r_1 = r_2$):

$$\delta_{\min} = 2i_1 - P$$

The index of refraction of the glass n from which a prism is made can be calculated by the formula

$$n = \frac{\sin \frac{1}{2}(\delta_{\min} + P)}{\sin \frac{1}{2} P}$$

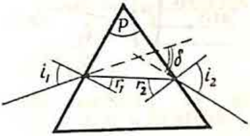


Fig. 73. Path of monochromatic rays in a prism

The maximum prism angle at which the rays still pass through the refracting faces is

$$P_{\max} = 2i_{\mathrm{cr}}$$

where i_{cr} is the critical angle of total reflection.

3. Lenses. Optical Instruments

A *lens* is a transparent body confined by two curved surfaces.

A lens is called *thin* if its thickness is considerably smaller than the radii of curvature of its surfaces.

The straight line passing through the centres of curvature of the surfaces of a lens is called its *principal optical axis*. If one of the surfaces of a lens is a plane, the optical axis is at right angles to it (Fig. 74).

The point of a thin lens through which rays pass without changing their direction is called the *optical centre* of the lens. The principal optical axis passes through the optical centre.

Any other straight line passing through the optical centre of a lens is called its *secondary axis*. The point at which rays of light that are parallel to the principal optical axis before entering a lens converge after passing through it is called its *focus*.

The plane passing through the focus at right angles to the principal optical axis is called the *focal plane*.

The formula of a thin lens (Fig. 75) is

$$-\frac{1}{a_1}+\frac{1}{a_2}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{1}{f} \qquad (5.5)$$

where a_1 is the distance from the lens to the object, a_2—the distance from the lens to the image, $-f_1 = f_2 = f$—the focal length of the lens, r_1 and r_2—the radii of curvature of the spherical surfaces confining the lens, and n—the relative index of refraction of the lens material.

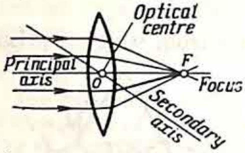


Fig. 74. Principal elements of a thin lens

In formula (5.5) the quantities a_1, a_2, r_1, and r_2 are considered to be positive if their directions when measured from the optical centre of the lens coincide with the direction of propagation of the light; otherwise these quantities are considered to be negative.

Lenses are the principal element of many optical instruments.

The eye, for example, is an optical instrument where the part of a lens is played by the cornea and the crystalline lens, while the image of an object is obtained on the retina.

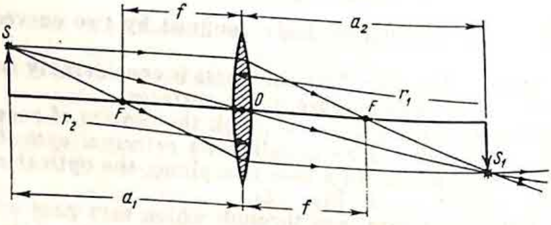


Fig. 75. Path of rays in a thin converging lens

The *angle of vision* is the angle formed by the rays passing from the extreme points of an object or its image through the optical centre of the crystalline lens of the eye.

Many optical instruments are intended for obtaining the images of objects on a screen, on light-sensitive films, or in the eye.

The visible magnification of an optical instrument is

$$\Gamma = \frac{\tan\varphi_2}{\tan\varphi_1}$$

where φ_2 is the angle of vision when observing the object through the instrument, and φ_1—the angle of vision when observing with the naked eye the object, which should be at a distance of 25 cm from the eye (for a magnifying glass and a microscope) or at the same distance as when looking through the instrument (for telescopes).

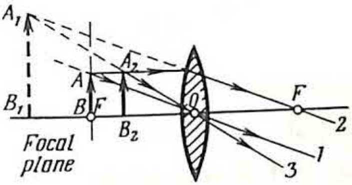


Fig. 76. Path of rays in a magnifying glass.
Object AB is in focal plane F. The image is obtained at infinity (rays 1 and 2 are parallel). Such an arrangement is also possible when the object is near focal plane A_2B_2. Image A_1B_1 will be virtual (on the continuation of rays 2 and 3); it will be at a distance from the eye that is the most convenient for viewing it

The lens in an optical instrument facing the object is called the *objective*, and the lens facing the eye—the *ocular*, or *eyepiece*. In technical instruments the objective and eyepiece consist of several lenses. This often eliminates errors in the images.

The magnification of a magnifying glass (Fig. 76) is

$$\Gamma_{\mathrm{m.g}} = \frac{250}{f} \qquad (5.6)$$

where f is the focal length of the magnifying glass in mm.

The magnification of a microscope (Fig. 77) is

$$\Gamma_{\mathrm{m}} = \frac{\Delta}{f_{\mathrm{obj}}}\,\frac{250}{f_{\mathrm{eye}}} \qquad (5.7)$$

where f_{obj} and f_{eye} are the focal lengths of the objective and the eyepiece in mm, respectively, and Δ—the distance from

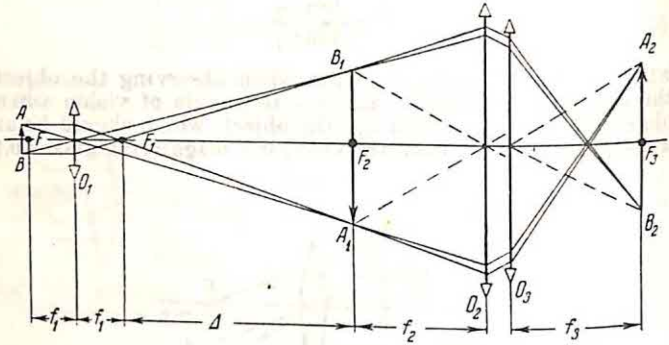


Fig. 77. Path of rays in a microscope.

AB is the object; Q_1—the objective of the microscope, which produces a magnified inverted and real image of object A_1B_1. The image of the object is in focal plane F_2 of the microscope eyepiece O_2 through which it is observed like through a magnifying glass. A real image of object A_2B_2 is obtained in focal plane F_3 of the crystalline lens O_3 of the eye. Such an arrangement of O_1 and O_2 is also possible when A_1B_1 is between F_2 and O_2

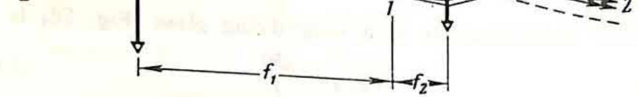


Fig. 78. Path of rays in a telescope.

Rays *1*, *1*, *1* impinge on objective O_1 from one point of a remote object in a parallel beam. The image of this point *1* is obtained in focal plane F of objective O_1 and eyepiece O_2. Rays *2*, *2*, *2* from a second point of the remote object also impinge on the objective in a parallel beam (but at a different angle to the optical axis). These rays also give an image of point *2* in focal plane F. The image of points *1*, *2* is observed through the eyepiece like through a magnifying glass the rear focus of the objective to the front focus of the eyepiece in mm.

The magnification of a telescope (Fig. 78) is

$$\Gamma_t = \frac{f_{obj}}{f_{eye}} \tag{5.8}$$

where f_{obj} and f_{eye} are the focal lengths of the objective and the eyepiece, respectively.

The reciprocal of the focal length is called the *power* of a lens:

$$D = \frac{1}{f}$$

The unit of power of a lens is the diopter (D) equal to the power of a lens with a focal length of 1 m.

The power of two thin lenses combined together is equal to the sum of their powers.

4. Wave Properties of Light

Interference. When two (or more) waves travel simultaneously in a medium, at each point of the medium the particles simultaneously participate in two (or more) oscillatory motions. The resulting displacement of a particle is determined by the rules of summation of oscillations (see p. 106). In the propagation of several electromagnetic waves the oscillations of the vectors of the electric and magnetic field intensities of the waves E and H at each point of space are summated like vectors (see p. 17).

The summation of two (or several) waves having identical periods leading to an increase in the amplitude of the resultant wave in some points of space and to diminishing of the amplitude in other points is called *interference*.

For example, if two plane electromagnetic waves with identical amplitudes and directions of propagation

$$E_1 = E_0 \sin(\omega t - kx_1)$$
$$E_2 = E_0 \sin(\omega t - kx_2)$$

have travelled the paths x_1 and x_2 from their source to the point of their observation, the resultant wave is determined by the expression

$$E_r = E_1 + E_2 = 2E_0 \cos \frac{k(x_2 - x_1)}{2} \sin\left[\omega t - \frac{k(x_2 + x_1)}{2}\right]$$

When $\frac{k(x_2-x_1)}{2}=\frac{1}{2}m\pi$ (where $m=1, 3, 5, \ldots$), the value of E_r equals zero; the difference in the paths of the waves $x_2-x_1=\frac{1}{2}m\lambda$, i.e. equals an odd number of half-waves.

If light travels different paths x_1 and x_2 in media having different indices of refraction n_1 and n_2, then upon summation of the waves the minimum value of the amplitude of the resultant wave will be observed at $x_2n_2-x_1n_1=\frac{1}{2}m\lambda$ (where m is an odd integer).

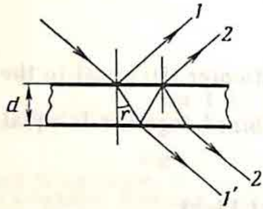


Fig. 79. Interference of rays impinging on a plane-parallel plate

The product of the geometrical path and the index of refraction of the medium is called the *optical path*.

The interference of any waves (including light waves) appears only when the interacting waves have the same frequency and a phase difference at each point that does not depend on time. Sources emitting such waves are called *coherent*. For the interference of polarized waves (see p. 207) their planes of polarization must also coincide.

Coherent sources can be created only artificially in optics.

The colours of thin films when illuminated by natural light depend on the interference of rays *1* and *2* (in reflected light), or of rays *1'* and *2'* (in transmitted light) (Fig. 79).

The optical difference in the paths of the interfering rays in this case is

$$\Delta=2dn\cos r+\frac{\lambda}{2}$$

where $\frac{\lambda}{2}$ takes into account the change in the wave phase by π for the vector of the electric field intensity when reflected from an optically more dense medium (at angles of incidence that do not exceed the angle of polarization, see below).

Diffraction. The deflection of light rays from a rectilinear path when passing near the edges of screens and openings is called *diffraction*.

The intensity of waves in diffraction phenomena can be approximately calculated with the aid of the *Huygens-Fresnel principle*. According to this principle the points of a wave surface (p. 109) are centres of imaginary coherent sources which emit spherical waves. These waves are called *elementary*.

The enveloping surface of the elementary waves is a wave surface for a following moment of time; it appears as a result of interference of the elementary waves.

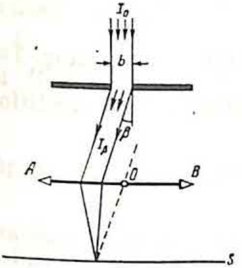


Fig. 80. Diffraction of parallel rays by a single slit.

AB—lens, *O*—optical centre of lens, *S*—screen in focal plane of lens

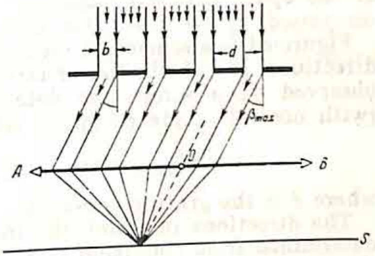


Fig. 81. Diffraction of parallel rays by a diffraction grating.

AB—lens, *O*—optical centre of lens, *S*—screen in focal plane of lens

It is assumed that the enveloping surface travels only in one direction from the wave surface (in the direction of propagation of the wave).

In the diffraction of a parallel beam of light impinging normally on a long narrow slit, the intensity I_β of the waves propagating at the angle β is determined by the relation

$$I_\beta = I_0 \sin^2\left(\frac{\pi b}{\lambda}\sin\beta\right) \Big/ \left(\frac{\pi b}{\lambda}\sin\beta\right)^2$$

where I_0 is the intensity of the waves travelling in the direction $\beta = 0$, b—the width of the slit, β—the angle between a normal to the plane of the slit and the direction of propagation of the diffracted rays (Fig. 80), and λ—the wavelength of the light.

The directions in which the intensity of the diffracted light from one slit equals zero are determined from the condition

$$\sin \beta_{\min} = m\frac{\lambda}{b} \qquad (m = 1, 2, 3, \ldots) \tag{5.9}$$

A group of a large number of narrow parallel slits having the same width and arranged in a plane at equal distances from one another is called a plane *diffraction grating*. The distance equal to the sum of the width of one slit and that of the space between adjacent slits is called the *constant* (or *period*) *of the diffraction grating*.

Figure 81 is a schematic view of a diffraction grating. The directions in which the maximum illumination will be observed on a screen are determined from the condition (with normal incidence of the light)

$$d \sin \beta_{\max} = m\lambda \qquad (m = 0, 1, 2, \ldots) \tag{5.10}$$

where d is the grating constant.

The directions in which the intensity will equal zero are determined from condition (5.9). These directions are called the *principal minima*.

Apart from these directions there is also a number of others in which the intensity of the diffracted light equals zero. The directions for the additional minima are determined from the condition

$$\sin \beta_{\mathrm{a.m}} = m' \frac{\lambda}{d}$$

$$\left(m' = \frac{1}{N}, \frac{2}{N}, \ldots, \frac{N-1}{N}, \frac{N+1}{N}, \ldots \right)$$

where N is the number of slits in the grating.

The interval of wavelengths resolved by a diffraction grating ($\Delta\lambda$) is determined from the condition

$$\frac{\lambda}{\Delta\lambda} = mN$$

The diffraction of light waves does not allow us to distinguish the most minute details of objects with the aid of optical instruments, no matter how great their magnification is.

The smallest distance between two points at which their images do not merge is called the *least resolved distance* of an optical instrument (δ).

For a microscope the least resolved distance is

$$\delta \geqslant \frac{\lambda}{2n \sin u} \tag{5.11}$$

where u is the *aperture angle* (half the angle subtended by the extreme rays from a point on the object which enter the objective and reach the observer's eye or a screen), and n—the index of refraction of the medium.

Dispersion of light. The velocity of light in a given medium depends on the wavelength. This phenomenon is called *dispersion of light velocity*.

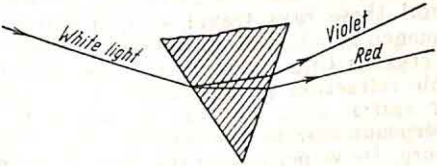


Fig. 82. Path of rays having different wavelengths in a glass prism

The index of refraction also varies with the wavelength (*dispersion of the index of refraction*).

Owing to dispersion a glass prism separates from white light, consisting of radiation of different wavelengths, its components, which after refraction travel in different directions. Rays having shorter wavelengths are refracted to the base of the prism through a greater angle than rays having greater wavelengths (Fig. 82).

Polarization of light. In the light waves emitted by different sources the vectors E (and, hence, H) are oriented randomly (however, they remain mutually perpendicular and the plane containing them is perpendicular to the direction of propagation of the light). Such light is called *natural*.

It is possible to separate from natural light such waves in which the vector E oscillates in the same plane along the entire path travelled by the waves. This can be done, for example, by passing the light through a tourmaline plate (or through a polaroid). These waves are called *linearly polarized*.

The plane containing the vectors H in a linearly polarized wave is called the *plane of polarization*; and the plane containing the vectors E—the *plane of vibration.*

Natural light when reflected from the surface of a dielectric becomes partly polarized. The reflected light will be polarized completely at an angle of incidence determined by the relation

$$\tan i_B = n \tag{5.12}$$

where n is the relative index of refraction of the dielectric from which the light is reflected. The angle i_B is called the *Brewster angle.*

Light passing through a dielectric is also partly polarized.

When light passes through some crystals (for example, quartz, Iceland spar), two rays emerge from them that are linearly polarized in two mutually perpendicular planes; in the crystal these rays travel with different velocities. This phenomenon is called *birefringence*, or *double refraction.*

Uniaxial crystals (like quartz) have only one direction in which double refraction is not observed. This direction is called their *optical axis.* The ray in which the vector E oscillates perpendicular to the optical axis of a crystal is called *ordinary*. Its velocities are the same in different directions in the crystal.

The second ray, for which the directions of oscillations of the vector E form different angles with the optical axis (depending on the angle and position of the plane containing the incident light), is called *extraordinary.*

The velocity of an extraordinary ray depends on its direction in a crystal. It may be greater than the velocity of the ordinary ray (as in Iceland spar) or smaller than it (as in quartz). The velocities of an ordinary and an extraordinary rays are equal in the direction of the optical axis.

Double refraction in isotropic media can be caused artificially by deformation or superposition of an electric and a magnetic fields. In this case the medium acts on the light like a uniaxial crystal with its optical axis parallel to the direction of deformation (or the field).

The difference between the optical paths (for an ordinary and an extraordinary rays) in deformation is

$$\Delta_P = KPl\lambda$$

where K is a constant depending on the nature of the substance, P—the mechanical stress, l—the thickness of the deformed layer, and λ—the wavelength of the light.

The difference between the optical paths for both rays when an electric or a magnetic field is superposed on a substance at right angles to the direction of propagation of the light is

$$\Delta_E = 1.11 \times 10^{-5} BE^2 l\lambda \quad \text{and} \quad \Delta_H = 1.58 CH^2 l\lambda$$

where B and C are the Kerr and Cotton-Mouton constants, respectively, E—the intensity of the electric field (V/cm), H—the intensity of the magnetic field (A/cm), and l—the thickness of the layer of the substance through which the light passes (cm).

Some substances (for example, quartz, a solution of sugar in water) when linearly polarized light passes through them turn its plane of polarization. This phenomenon is called *rotation of the plane of polarization.*

The rotation of the plane of polarization is characterized by the rotational constant (for pure substances) and the unit rotational constant (for solutions).

The *rotational constant* $\alpha = \frac{\varphi}{l}$ and the *unit rotational constant* $[\alpha] = \frac{100\,\varphi}{lp'}$, where φ is the angle of rotation of the plane of polarization, l—the thickness of the layer of optically active substance, and p'—the number of grammes of solute in 100 cm^3 of the solution. The value of $[\alpha]$ depends on the concentration, temperature, wavelength (rotational dispersion), and the solvent.

Pressure of light. When electromagnetic waves impinge on the surface of a body, they exert mechanical pressure on it (called the *radiation pressure*).

The magnitude of the radiation pressure is given by the formula

$$p = \frac{W}{c}(1+\rho) \tag{5.13}$$

where W is the quantity of radiation energy impinging normally on 1 cm^2 of surface per second, c—the velocity of light, and ρ—the reflection coefficient.

The pressure exerted by the Sun's rays on the Earth's surface on a bright day is equal approximately to 0.4 dyn/m^2 (4×10^{-5} dyn/cm^2).

5. Quantum Properties of Light

The energy of any form of electromagnetic radiation, including light, always exists in the form of discrete portions. These portions of energy, which possess the properties of material corpuscles, are called *radiation quanta* or *photons*. A photon is an elementary particle (see p. 237). The energy of a photon ε depends on the frequency of the radiation ν:

$$\varepsilon = h\nu$$

where $h = 6.625 \times 10^{-27}$ erg·s is called *Planck's constant*.

According to the fundamental principles of modern physics whenever the energy of a system changes by an amount ε there is an equivalent change in the mass of the system equal to $\frac{\varepsilon}{c^2}$ (c is the velocity of light in a vacuum). Hence, for every photon emitted by a body its mass decreases by an amount

$$\Delta m = \frac{h\nu}{c^2}$$

Those properties of light which are due to the discrete nature of radiation are called *quantum* (or *corpuscular*) *properties.*

Light, like all other forms of electromagnetic radiation, possesses both wave and corpuscular properties.

The term *photoelectric effect* is applied to electrical phenomena occurring when a substance is illuminated with light, namely, the emission of electrons from the substance (*photoelectron emission*), the appearance of an e.m.f., a change in electric conductivity.

The photoelectric effect is an example of the manifestation of the corpuscular nature of light. The emission of electrons from illuminated bodies is called the *extrinsic photoeffect* or the *photoemissive effect.*

Laws of the photoemissive effect. 1. The number of electrons liberated by light in one second (or the saturation current) is directly proportional to the light flux (the spectral composition of the light remaining constant).

2. The maximum velocity of the emitted electrons v_{max} does not depend on the light intensity, but is determined by

the frequency of the incident light. This velocity can be found by *Einstein's equation*

$$h\nu = \varphi + \frac{mv_{\max}^2}{2} \tag{5.14}$$

where $h\nu$ is the energy of a photon, φ—the work function (see p. 143), and m—the mass of an electron.

3. For every substance there is a definite frequency below which the photoelectric effect is not observed. This frequency is called the *photoelectric threshold* (ν_{thr}). It is determined from the relation

$$h\nu_{\text{thr}} = \varphi \tag{5.15}$$

When semiconductors and dielectrics are illuminated, some of their atoms may lose electrons, which, however (in contrast to the photoemissive effect), do not escape through the surface of the body, but remain within it. This phenomenon is called the *intrinsic photoeffect* or the *photoconductive effect*. As a result of the photoconductive effect electrons appear in the conduction band (p. 145), and the resistance of the semiconductors and dielectrics diminishes.

When the interface between semiconductors having different kinds of conductance is illuminated, an electromotive force appears. This phenomenon is called the *barrier-layer photoeffect*.

The design of photoelectric cells, photoconductive cells, barrier-layer cells, and solar batteries is based on the phenomena of the photoelectric effect.

6. Types of Spectra

Any oscillation (a wave or a pulse) can be represented as the result of summation (superposition) of harmonic oscillations (called harmonics for purposes of brevity) having different amplitudes and frequencies (see p. 103). The aggregate of amplitudes and frequencies of the harmonics which a complex oscillation (or wave) consists of is called a *spectrum of amplitudes and frequencies* (or more briefly a *spectrum*).

The coloured bands obtained when light is broken up by a prism (or other device) according to wavelength is sometimes called a spectrum (in the narrow sense of the word).

Heated solids emit a *continuous spectrum* in which all the spectral colours are present, one colour gradually merging into another.

In *line spectra* the intensity of radiation is high only within very narrow regions contiguous to definite wavelengths, while at other wavelengths it is practically zero.

Line spectra are emitted by gases of low density. If such a spectrum is dispersed by a prism (or other instrument), narrow lines of different colours are obtained. The number of these lines and their wavelengths are characteristic for each element. An analysis of the lines in the spectrum thus allows us to identify the elements which are present in the given substance.

The molecules of a substance produce a spectrum consisting of a great number of closely arranged lines grouped into separate bands. Such spectra are called *band spectra*.

The spectra of heated bodies are called *emission spectra*.

The dependence of the absorptivity of a substance on the wavelength of light is called an *absorption spectrum*. The latter may be continuous, band, and line.

In the emission spectra of sources of light whose internal zones are surrounded by colder layers of vapour, narrow absorption lines are observed. They are due to the absorption of the light by the vapour in the outer layers. Gases greatly absorb such portions of the spectrum which they themselves emit being sources of light (the *Kirchhoff-Bunsen law*). This phenomenon is called the *reversal of spectrum lines*.

The *Fraunhofer lines* in the solar spectrum are absorption spectrum lines appearing as a result of the absorption of radiation of definite wavelengths from the continuous solar spectrum by vapours present in the atmosphere.

7. Thermal Radiation

Heated bodies emit invisible waves—the so-called *ultraviolet* and *infrared waves*, in addition to light waves. The wavelength of ultraviolet waves (see Table 117) is less than 4000 Å, and that of infrared waves exceeds 8000 Å.

The electromagnetic radiation generated by the thermal motion of atoms or molecules is called *thermal* or *heat radiation*.

The distribution of the emissive power (see below) in the radiation spectrum of a black body is shown in Fig. 83. An oblique-angled system of coordinates with logarithmic scales has been used to plot the graph. The wavelengths are plotted along the horizontal axis, and the emissive power along the inclined one; the figures on the curves indicate the absolute temperature.

A body which completely absorbs all the radiation incident on it is called a *black body* (or a *perfect absorber*). Such a body can be made in the form of a hollow box with a small aperture.

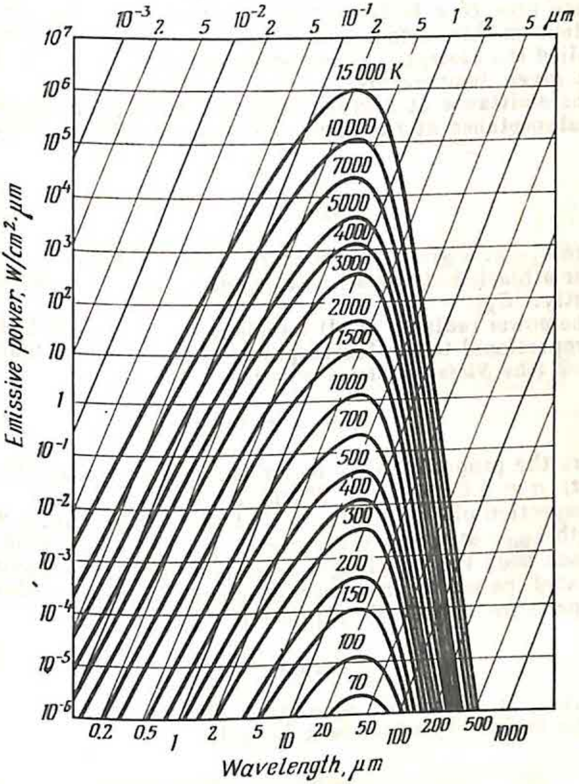


Fig. 83. Radiation of black body at different temperatures

The ratio of the radiated power from a unit of surface area to the corresponding interval of its wavelengths at a given temperature is called the *emissive power* or *emittance* of a body $E_{\lambda T}$.

The emittance equals the power radiated from a unit of surface area per unit of wavelength interval at a given temperature.

The ratio of the radiated power absorbed by a unit of surface area of a body to the power of the radiation flux incident on this surface with a unit interval of wavelengths is called the *absorptive power* or *absorptance* $A_{\lambda T}$ of the body at a given temperature.

The emittance at a given temperature is proportional to the absorptance at the same temperature (*Kirchhoff's law*):

$$\frac{E_{\lambda T}}{A_{\lambda T}} = \varepsilon_{\lambda T} \tag{5.16}$$

where $\varepsilon_{\lambda T}$ at a given temperature is constant for all bodies

For a black body at all wavelengths $A_{\lambda T} = 1$ and, consequently, $E_{\lambda T} = \varepsilon_{\lambda T}$.

The power radiated at all wavelengths by a black body (ε) is proportional to the fourth power of the absolute temperature T (the *Stefan-Boltzmann law*):

$$\varepsilon = \sigma T^4 \tag{5.17}$$

where the proportionality factor (the *Stefan-Boltzmann constant*) $\sigma = 5.67 \times 10^{-12}$ W/cm$^2 \cdot$K^4.

Inspection of Fig. 83 shows that there is a definite wavelength $\lambda_{\max}$ at which the radiated power is maximum. For a black body the wavelength corresponding to the maximum radiated power is inversely proportional to the absolute temperature T (*Wien's displacement law*):

$$\lambda_{\max} T = c_1 \tag{5.18}$$

where c_1 is a constant equal to 0.2898 cm$\cdot$K.

The emissivity of a black body is

$$\varepsilon_{\lambda T} = \frac{2\pi c^2}{\lambda^5} \frac{h}{e^{hc/(k\lambda T)} - 1} \tag{5.19}$$

Equation (5.19) is called *Planck's formula*; here c is the velocity of light in a vacuum, λ—the wavelength, k—the

Boltzmann constant, and h—the Planck constant. The function (5.19) is shown graphically in Fig. 83 (for different temperatures).

TABLES AND GRAPHS

Table 110

Relative Luminosity K_λ for Daytime Vision (see Fig. 84)

Wave-length, Å	K_λ	Wave-length, Å	K_λ	Wave-length, Å	K_λ
4000	0.0004	5200	0.710	6400	0.175
4200	0.0040	5400	0.954	6600	0.061
4400	0.023	5600	0.995	6800	0.017
4600	0.060	5800	0.870	7000	0.0041
4800	0.139	6000	0.631	7200	0.00105
5000	0.323	6200	0.381	7400	0.00025
				7600	0.00006

Note. The values of the relative luminosity vary for different people. They do not vary very greatly, however, for people with normal vision. The table gives the average values of K_λ.

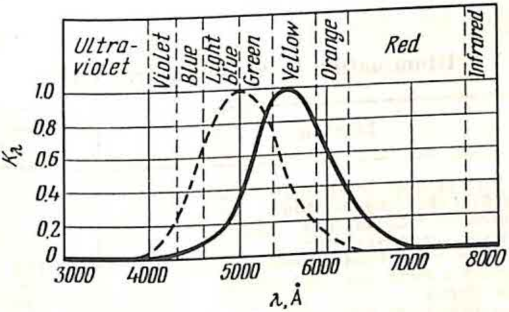


Fig. 84. Graph of relative luminosity for daytime and twilight (dash curve) vision

Table 111

Luminance of Illuminated Surfaces

Illuminated surface	Luminance, nt
Cinema screen	5-20
Sheet of white paper (under illuminance of 30-50 lux)	10-15
Snow in direct sunlight	3×10^4
Moon's surface	2.5×10^3

Table 112

Luminance of Various Light Sources

Source	Luminance, nt
Sun	15×10^8
Capillary of superhigh pressure mercury arc	12×10^8-15×10^8
Carbon arc crater	15×10^7
Metal filament of incandescent lamp	1.5×10^6-2×10^6
Moonless night sky	10
Spark discharge in xenon	1.1×10^{11}
Ditto in argon	1.5×10^{11}
Ditto in air (nitrogen)	2.1×10^{11}
Ditto in helium	1.5×10^{12}

Table 113

Illumination in Some Typical Cases

Lighting	Illumination, lx
Sun's rays at noon (middle latitudes)	100 000
During film shooting in studio	10 000
Open area on a cloudy day	1 000
Light room (near window)	100
Work table for delicate operations	100-200
Illumination necessary for reading	30-50
Cinema screen	20-80
Illumination from full moon	0.2
Illumination from moonless night sky	0.0003

Table 114

Reflection Coefficient (ρ, %) for Glass and Water at Different Angles of Incidence

Substance \ Angle of incidence, degrees	0	20	30	40	50	60	70	80	89	90
Glass	4.7	4.7	4.9	5.3	6.6	9.8	18	39	91	100
Water	2	2.1	2.2	2.5	3.4	6.0	13.5	34.5	90.0	100

Note. For glass covered with a film of silica having an index of refraction of 1.5, $\rho \approx 2.5$ at normal incidence. For glass with a film of silica having an index of refraction of 1.9, $\rho \approx 0.8$ at normal incidence.

Table 115

Reflection of Light Passing from Glass into Air

Angle of incidence	0°	10°	20°	30°	35°	39°	39°30′	40°	60°
Angle of refraction	0°	15°40′	32°	51°	63°	79°	82°	90°	—
Fraction of reflected energy in %	4.7	4.7	5.0	6.8	12	36	47	100	100

Note. The table has been compiled for glass having an index of refraction of 1.56, for which the maximum angle of complete reflection is 40°.

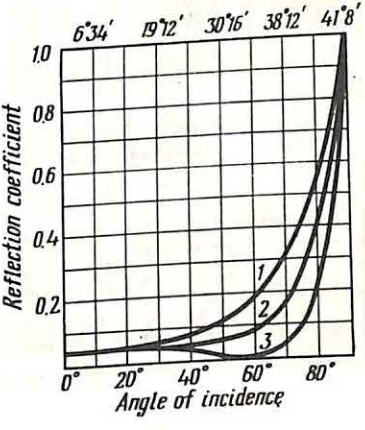


Fig. 85. Dependence of coefficient of reflection of light on angle of incidence at air-glass interface ($n = 1.52$): 1—reflection of plane-polarized waves (the direction of the oscillations of the vector E is perpendicular to the plane of incidence); 2—reflection of non-polarized light; 3—reflection of plane-polarized waves (the vector E is in the plane of incidence). The figures at the top are the angles of incidence when light passes from glass to air

Table 116

Wavelengths of Visible Region of Spectrum

Colour	Boundaries, Å	Colour	Boundaries, Å
Violet	3800-4500	Yellowish-green	5500-5750
Dark blue . . .	4500-4800	Yellow	5750-5850
Light blue . . .	4800-5100	Orange	5850-6200
Green	5100-5500	Red	6200-7600

Table 117

Wavelengths of Ultraviolet Region of Spectrum

Range	Boundaries, Å	Effect of radiation
Long-wave ultraviolet	3150-3800	Suntan
Middle-wave ultraviolet . . .	2800-3150	Erythemogenic
Short-wave ultraviolet	2000-2800	Bactericidal
Vacuum ultraviolet	< 2000	Ozonizing

Table 118

Reflection of Light by Metals

Name of wave group	Wavelength, Å	Aluminium	Copper	Steel (1% C)	Silver	Nickel	Chromium	Zinc	Silicon
Ultraviolet	1 880	—	—	22	76	—	33	17	64
	2 000	—	—	27	76	—	36	22	73
	2 510	80	26	38	34	38	—	39	75
	3 050	—	25	44	9	44	37	48	73
	3 570	84	27	50	75	49	41	51	60
Visible light	5 000	88	44	56	91	61	55	55	34
	6 000	89	72	57	93	65	—	58	32
	7 000	87	83	58	95	69	56	61	—
Infrared	8 000	85	89	61	97	70	—	62	—
	10 000	93	90	63	97	72	57	69	—
	50 000	94	98	90	99	94	81	97	—
	100 000	97	98	94	99	—	93	—	—

Note. The figures in the table indicate the fraction of the light (in %) reflected by a polished surface (with normal incidence).

Table 119

Critical Angles of Reflection (in degrees)

Substance	i_{cr}	Substance	i_{cr}
Water	49	Carbon disulphide . .	38
Glycerine	43	Glass (heavy flint) . .	34
Glass (light crown) .	40	Diamond	24

Note. The values of i_{cr} given in the table are for an interface with air (for the *D* line of sodium, 5893 Å).

Table 120

Wavelengths of Principal Fraunhofer Lines

Line	Element	λ, Å	Line	Element	λ, Å
A	Oxygen	7621	b_4	Iron, magnesium	5167.4
a	Ditto	7185	c	Ditto	4957.6
B	Ditto	6870	*F*	Hydrogen . . .	4861.3
C	Hydrogen . . .	6582.8	*d*	Iron	4668
α	Oxygen	6278.1	*e*	Ditto	4383.6
D_1	Sodium	5895.9	*f*	Hydrogen . . .	4340.5
D_2	Ditto	5890.0	*G'*	Iron	4325.8
D_3	Helium	5875.6	*G*	Iron, calcium	4307.9
E	Iron	5269.6	*g*	Calcium	4226.7
b_1	Magnesium . .	5183.6	*h*	Hydrogen . . .	4101.7
b_2	Ditto	5172.7	*H*	Calcium	3963.6
b_3	Iron	5169.0	*k*	Ditto	3933.6

Table 121

Index of Refraction for Wavelengths Corresponding to Selected Fraunhofer Lines

	Fraunhofer line	*A*	*B*	*D*	*F*	*H*
	Wavelength, Å	7590	6870	5890	4860	3970
Substance	Carbon disulphide . .	1.610	1.617	1.629	1.654	1.702
	Ethyl alcohol	1.359	1.360	1.363	1.367	1.374
	Glass (light crown) .	1.510	1.512	1.515	1.521	1.531
	Water	1.329	1.331	1.333	1.337	1.344

Table 122

Index of Refraction of Gases

Gas or vapour	n	Gas or vapour	n
Acetylene	1.000 606	Hydrogen sulphide	1.000 641
Air	1.000 292	Mercury	1.000 933
Ammonia	1.000 377	Methane	1.000 441
Benzene	1.001 812	Nitrogen	1.000 297
Carbon dioxide	1.000 450	Oxygen	1.000 272
Carbon tetrachloride	1.001 763	Selenium	1.001 565
Chloroform	1.001 455	Sulphur dioxide	1.000 737
Helium	1.000 035	Tellurium	1.002 495
Hydrogen	1.000 138	Water vapour	1.000 257
		Zinc	1.002 050

Note. The values of the indices of refraction given in the table refer to a wavelength corresponding to the yellow line of sodium (D) and have been reduced to the density corresponding to 0 °C and a pressure of 760 mm Hg with the aid of the relationship $\frac{(n-1)}{\rho} = \text{const}$ (for a given gas).

Table 123

Index of Refraction of Selected Solids and Liquids (at 15 °C for the D-Line of Sodium Relative to Air)

Substance	n	Substance	n
Solids		Ethyl ether	1.354
Diamond	2.417	Glycerine	1.47
Ice	1.31	Methyl alcohol	1.33
Mica	1.56-1.60	Sulphuric acid	1.43
Sugar	1.56	Turpentine	1.47
Topaz	1.63	Water (20 °C)	1.333
Liquids		*Oils*	
Aniline	1.590	Cedar nut	1.516
Benzene	1.504	Cinnamon	1.601
Canada balsam	1.53	Clove	1.532
Carbon disulphide	1.632	Olive	1.46
Chloroform	1.449	Paraffin	1.44
Ethyl alcohol	1.362		

Table 124

Dependence of Index of Refraction on Wavelength

Wavelength region	Wavelength, Å	Glass (15 °C)		Quartz (18 °C)		Fused quartz	Water at 20 °C	Sylvite (KCl) at 18 °C
		light crown	heavy flint	ordinary ray	extraordinary ray			
Infrared	223 000	—	—	—	—	—	—	1.3712
	94 290	—	—	—	—	—	—	1.4587
	42 000	—	—	1.4569	—	—	—	1.4720
	21 720	1.4946	1.6153	1.5180	1.5261	—	—	1.4750
	12 560	1.5042	1.6268	1.5316	1.5402	—	1.3210	1.4778
Visible	6 708	1.5140	1.6434	1.5415	1.5505	1.4561	1.3308	1.4866
	6 438	1.5149	1.6453	1.5423	1.5514	1.4568	1.3314	1.4877
	5 893	1.5170	1.6499	1.5443	1.5534	1.4585	1.3330	1.4904
	4 864	1.5230	1.6637	1.5497	1.5590	1.4632	1.3371	1.4983
	4 047	1.5318	1.6852	1.5572	1.5667	1.4697	1.3428	1.5097
Ultraviolet	3 034	1.5552	—	1.5770	1.5872	1.4869	1.3581	1.5440
	2 144	—	—	1.6305	1.6427	1.5339	1.4032	1.6618
	1 852	—	—	1.6759	1.6901	1.5743	—	1.8270
Temperature coefficient, K^{-1}		-1×10^{-6}	3×10^{-6}	-5×10^{-6}	-6×10^{-6}	-3×10^{-6}	-8×10^{-5}	-4×10^{-5}

Note. 1. The refractive indices are given relative to air.

2. The *temperature coefficient* is the increment in the index of refraction per 1 K rise in temperature. The minus sign denotes that the refractive index decreases with a rise in temperature.

3. A light ray passing through a quartz crystal is split up into two polarized rays. The table gives the indices of refraction for both rays. The maximum values are indicated for the extraordinary ray.

Table 125

Diffuse Reflection of Selected Materials in White Light

Material	Reflection, %	Material	Reflection, %
Cardboard, white	60-70	Paper, brown	13
Ditto, yellow	30	Ditto, chocolate colour	4
Cheesecloth	16	Ditto, light blue	25
Fatty clay (yellow)	24	Ditto, ordinary white	60-70
Materials coated with white paint	50	Ditto, yellow	25
Materials coated with yellow paint	40	Tracing paper	22
Moist earth	8	Velvet, black	0.4
Paper, blotting	70-80	Wood (pine)	40

Table 126

Kerr and Cotton-Mouton Constants

Substance	B, 10^{-7}	C, 10^{-13}
Benzene	0.60	7.5 (26 °C, 5800 Å)
Carbon disulphide	3.21	−4.0 (28 °C, 5400 Å)
Chloroform	−3.46	−0.8 (20 °C, 5460 Å)
Water	4.7	−0.014 (20 °C, 5460 Å)
Chlorobenzene	10.0	10.2 (25 °C, 5460 Å)
Nitrotoluene	123	
Nitrobenzene	220	24.1 (20 °C, 5890 Å)

Notes. 1. The values of B are indicated for a temperature of 20 °C and a wavelength of sodium of 5890 Å.

2. The minus sign indicates that the velocity of a beam, the direction of whose oscillations coincides with that of the field, will be greater than the velocity of a beam, the direction of whose oscillations is perpendicular to the direction of the field.

Table 127

Unit Rotational Constant $[\alpha]$ at 20 °C (in angular degrees/dm)

Substance	Solvent	$[\alpha]$ ($\lambda = 5893$ Å)	Concentration p'
Cane sugar . .	Water	$66.47+1.27\times10^{-2}\,p' - 3.8\times10^{-4}\,(p')^2$	0-50
Glucose	Ditto	$52.50+1.88\times10^{-2}\,p' + 5.17\times10^{-4}\,(p')^2$	0-35
Tartaric acid	Ditto	$14.83-0.146\,p'$	0-50
Turpentine . .	Pure	-37	—
Camphor . . .	Alcohol	$40.9+0.135\,p'$	10-50
Quinine	Ditto	-169	2.0
Mandelic acid	Water	$+156$	3.2

Note. The minus sign indicates that rotation is counterclockwise when looking at the substance against the direction of the beams.

Table 128

Rotary Dispersion at 20 °C

Cane sugar in water (p'=10-26)		Tartaric acid in water (p'= 28.62)		Quartz (along optical axis)	
λ, Å	$[\alpha]$	λ, Å	$[\alpha]$	λ, Å	$[\alpha]$
3000	309	2750	−296.8	1854	370.9
4861	100.3	3000	−166.0	1935	322.8
5351	81.8	3500	−16.8	2571	143.3
5893	66.5	4500	6.6	4340	41.92
6563	52.9	5500	8.4	4861	32.76
		5890	9.8	5893	21.72
				6563	17.32

Note. When the temperature rises by 1 kelvin, the value of $[\alpha]$ grows by 0.003 of the original value (for $\lambda = 5893$ Å).

Table 129

Emission Spectra of Metals and Gases

Aluminium (arc)	Copper (arc in vacuum)	Mercury (mercury lamp)	Sodium (in flame)	Cadmium (arc)	Zinc (arc in vacuum)
3083	3248	3126	5889.97 *o*	3261	3036
3093	3274	3131	5895.93 *o*	3404	3072
3944 *v*	4023 *v*	3650		3466	3345
3962 *v*	4063 *v*	4046.8 *v*		3611	4680 *b*
4663 *b*	5105.5 *g*	4358.3 *v*		3982 *v*	4722 *b*
5057 *g*	5153.3 *g*	4916.4		4413 *b*	4811 *b*
5696 *y*	5218.2 *g*	4959.7 *g*		4678 *b*	4912 *b*
5723 *y*	5700	5460.7 *g*		4799.9 *b*	4925 *b*
	5782.1 *y*	5769.6 *y*		5085.8 *g*	6103 *o*
	5782.2	5790.7 *y*		5338 *g*	6362 *o*
		6152.0 *o*		5379 *g*	
		6232.0 *o*		6438.5 *r*	

Nitrogen	Argon	Hydrogen	Helium	Oxygen	Neon
		(in gas discharge)			
5754	3949.0	4101.7	3888.6	5200	4537.8
5803	4044.4	4340.4	4026.2	5300	4575.9
5853	4158.6	4340.5	4120.8	(band)	4704.4
5904	4164.2	4861.28	5015.7	5550	4708.9
5957	4181.9	4861.36	5875.6	5640	4715.3
6012	4190	6562.71	6678.1	(band)	4788.9
6068	4191.0	6562.85	7065.2		5330.8
6251	4198.3				5341.1
6321	4200.7				5358.0
6393	4251.2				5400.6
6467	4259.4				5852.5
6543	4266.3				5881.9
6622	4272.2				5965.4
6703	4300.1				6143.1
6787	4333.6				6266.5
	4335.4				6383.0
					6402.2
					6506.5
					7173.9
					7245.2

Notes. 1. The wavelengths were measured in air at 15 °C and 760 mm Hg.

2. The colours of the visible lines are denoted by the first letter of the corresponding colour.

3. The brightest lines are underlined.

Table 130

Luminous Efficiency, Efficiency, and Luminance of Selected Light Sources

Type of lamp	lm/W	η, %	*t*, K	Luminance, nt
50-watt, carbon filament, vacuum . . .	2.5	0.4	2095	5×10^5
50-watt, tungsten filament, vacuum . . .	10	1.6	2460	15×10^5-20×10^5
50-watt, gas-filled, tungsten filament . .	10	1.6	2685	5×10^6
500-watt, gas-filled, tungsten filament . .	17.5	2.8	2900	10^7
2000-watt, gas-filled, tungsten filament . .	21.2	3.5	3020	13×10^6-15×10^6
Voltaic arc	25	4	4000	15×10^7 (in crater)
Luminescent lamp . .	40-60	6.4	—	1.5×10^4

Note. The *luminous efficiency* is the ratio of the total light flux to the power of the current feeding the light source.

The *efficiency* η is the ratio of the radiated power for the wavelength 5550 Å to the power used by the source of light:

$$\eta=\frac{F}{680W}$$

where F is the light flux in lm, and W—the power used by the source in W.

Table 131

Electron Work Function and Photoelectric Threshold

Substance	φ, eV	λ_{thr}, Å
Cuprous oxide	5.15	2 500
Mica	4.8	2 548
Sodium chloride	4.2	2 950
Silver bromide	3.7-5.14	3 350-2 400
Thorium on tungsten	2.62	4 730
Sodium on tungsten	2.10	5 900
Cesium on tungsten	1.36	9 090
Ditto on platinum	1.31	8 950
Barium on tungsten	1.1	11 300
Barium oxide on oxidized tungsten	1.0-1.1	12 400-11 300

CHAPTER VI

STRUCTURE OF THE ATOM AND ELEMENTARY PARTICLES

FUNDAMENTAL CONCEPTS AND LAWS

1. Units of Charge, Mass, and Energy in Atomic Physics

The unit of charge is the elementary charge

$$e = 1.60 \times 10^{-19} \text{ C}$$

The unit of mass is taken equal to $\frac{1}{12}$ of the mass of the carbon isotope (see p. 230) with an atomic mass of 12. This unit is called the *atomic mass unit* (amu);

$$1 \text{ amu} = 1.66 \times 10^{-24} \text{ g}$$

The mass of the lightest atom—the hydrogen atom—equals 1.008 amu.

The unit of energy is the electron-volt (eV), see p. 143;

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-12} \text{ erg}$$

The moments of momentum of particles (or the angular momenta) are measured in the units $\frac{h}{2\pi}$. The magnetic moments of electrons are measured in the units $\frac{eh}{4\pi mc}$;

this unit is called the *Bohr magneton*. In the latter expression h is the Planck constant, e—the charge of an electron, m—its mass, and c—the velocity of light in a vacuum. The magnetic moments of nuclei are measured in nuclear magnetons; a nuclear magneton equals $\frac{eh}{4\pi Mc}$, where M is the mass of a proton, and e is its charge.

The Bohr magneton $\mu_B = 9.273 \times 10^{-21}$ erg/Gs
The nuclear magneton $\mu_n = 5.050 \times 10^{-24}$ erg/Gs

2. The Rutherford-Bohr Model of the Atom

The centre of an atom consists of a positively charged nucleus around which electrons revolve in definite orbits. The mass of an atom is concentrated primarily in its nucleus.

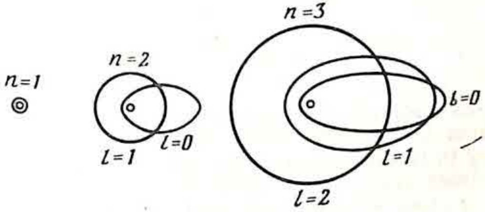


Fig. 86. Possible orbits of the electron in a hydrogen atom (for different quantum numbers). The semimajor axes of the ellipse are in the ratio of the squares of integers $1^2:2^2:3^2:4^2$, etc.

The nucleus of the hydrogen atom is called a *proton*. The mass of a proton is 1.66×10^{-24} g, and of an electron is 9.11×10^{-28} g $\left(\frac{1}{1836}\text{ of the mass of a proton}\right)$. The charge of an electron or a proton equals the elementary electric charge.

The number of elementary charges of a nucleus equals the atomic number of the relevant element in Mendeleev's Periodic Table. The number of electrons in a neutral atom equals the number of elementary charges of the nucleus.

The motion of the electrons in an atom can be approximately described as motion along definite circular or elliptical orbits around the nucleus (Fig. 86). These orbits are called *stationary*. When an electron revolves about the nucleus along a stationary orbit, it does not radiate, notwithstanding the fact that it has acceleration. The radii of the stationary orbits are determined from the condition

$$mv_nr_n = \frac{h}{2\pi}n \tag{6.1}$$

where m is the mass of an electron, v_n—its linear velocity, r_n—the radius of the n-th orbit, h—the Planck constant, and $n = 1, 2, 3, \ldots$.

Definite values of the energy (discrete energy levels) correspond to definite stationary orbits of the electrons (more exactly, to stationary states of the atom).

The energy levels (W_n) and the radii of the circular orbits of the hydrogen atom can be computed by the formulas:

$$W_n = -\frac{2\pi^2me^4}{n^2h^2} \tag{6.2}$$

$$r_n = \frac{n^2h^2}{4\pi^2me^2} \tag{6.3}$$

An atom radiates electromagnetic energy if an electron jumps from a stationary orbit farther from the nucleus to one closer to it; an atom absorbs energy if an electron jumps from a closer orbit to a farther one. The magnitude of the emitted or absorbed quantum (portion) of energy $h\nu$ is given by the condition

$$h\nu = W_2 - W_1 \tag{6.4}$$

where W_1 and W_2 are the discrete energy levels in the atom before and after the transition of the electron.

According to contemporary views, the stationary orbits do not represent the actual movements of an electron in an atom, and the structure of the atom is interpreted in a different way—with account taken of the wave properties of particles (see p. 234). The concept of the energy levels in an atom, however, remains in force.

3. Electron Shells and Many-Electron Atoms

The state of an electron in an atom is determined by four quantum numbers. The *main quantum number n* determines the energy levels (or the major axis of the ellipse). The second quantum number—the *orbital quantum number* l—is related to the semiminor axis of the ellipse and determines the shape of the orbit; l varies within the limits from 0 to $n-1$; when $l = n - 1$, the orbit is circular. The *angular momentum* (or *moment of momentum*) is determined by this number: $p = \sqrt{l(l+1)}\,\hbar$, where $\hbar = \frac{h}{2\pi}$. The third quantum number—the *magnetic quantum number* m—determines the orientation of the orbit in a magnetic field; the orbits are oriented so that the projections of the angular momentum p_z onto the direction of the magnetic field equal integers (in units of $\hbar$): $p_z = m\hbar$; m changes within the limits from $-l$ to $+l$.

Apart from the orbital angular momentum, an electron has its *intrinsic angular momentum* (*spin*) equal in magnitude to $\sqrt{s(s+1)}\,\hbar$, where $s = \frac{1}{2}$. This is the fourth quantum number, called the *spin quantum number*. The intrinsic angular momentum of an electron has only two projections onto the direction of the magnetic field: $+\frac{1}{2}$ and $-\frac{1}{2}$ (in units of $\hbar$). It is customary practice to say that the spin of an electron (or another particle) equals one-half $\left(\frac{1}{2}\right)$, it being understood that the projections of the intrinsic angular momentum equal $\pm\left(\frac{1}{2}\right)\hbar$.

The four quantum numbers determine the energy levels of an electron.

The electrons in an atom obey the *Pauli exclusion principle*: only one electron can be in a given state.

Upon the transition of an electron from one energy level to another the quantum numbers change. Theory and practice show that the transition of an electron is possible to such levels when the change in Δm equals 0 or ± 1, and the change in Δl equals ∓ 1.

In an atom the complex of electrons having identical quantum numbers n form a *layer* (or *group*); the complex of electrons having identical quantum numbers n and l form a *shell*.

The layers are denoted by the letters K, L, M, N, etc. (which corresponds to the values of the main quantum number $n = 1, 2, 3, 4$, etc.); the shells are denoted by the letters

s, p, d, f, etc. (which corresponds to the values of the orbital quantum number $l = 0, 1, 2, 3$, etc.).

The maximum number of electrons in a layer is $2n^2$. The distribution of electrons by layers and shells is given in Tables 132 and 133.

The chemical properties of an atom are determined by the electrons on its outer shell. Transitions of electrons in this shell create light and ultraviolet electromagnetic waves.

If an electron is removed from an inner shell of an atom, an electron will spontaneously transfer to this shell from other ones with greater numbers n. Such transitions are attended by the emission of line spectra of X-ray radiation called *characteristic*. The wavelengths of these spectra are determined only by the discrete levels of energy of atoms of the given substance.

4. The Atomic Nucleus

The atomic nucleus of any element consists of protons and neutrons. A *neutron* is a particle having no charge and with a mass close to that of a proton; the spin of a neutron, like that of a proton, equals $\left(\frac{1}{2}\right)\hbar$.

Nuclei having an identical charge (and an identical number of protons), but differing in the number of neutrons, are called *isotopes*. The sum of the neutrons and protons in a nucleus is called the *mass number* (A).

The number of neutrons in a nucleus is $N = A - Z$, where Z is the atomic number of the nucleus, equal to its atomic number in the periodic table. The charge of the nucleus equals Ze. Both particles—the proton and the neutron—are called *nucleons* (nuclear particles); they are bound together inside the nucleus by special forces, called *nuclear forces*. The nucleons in a nucleus are at discrete energy levels. Nuclei in which the number of neutrons or protons equals one of the following numbers: 2, 8, 20, 28, 50, 82, and 120 are called *magic nuclei*, and the numbers themselves—*magic numbers*. Such nuclei have a great stability.

The density of the substance in a nucleus is very great (of the order of 10^{11} kg/cm^3), the size of a nucleus is of the order of magnitude of 10^{-13} cm (1 fermi). The radius of a nucleus is determined by the approximate formula

$$r = 1.2A^{1/3} \text{ fermi}$$

where A is the mass number.

The spin of a nucleus J consists of the spins of the nucleons and their angular momenta; the spin of a nucleus has a half-integral value with an odd number of nucleons; with an even number of nucleons the spin of a nucleus equals zero or an integer. Nuclear spins do not exceed several units (in the units $\hbar$).

A nucleus is designated by the symbol of the relevant chemical element with the atomic number as a subscript and the mass number as a superscript (for example, Al_{13}^{27} stands for aluminium with an atomic number of 13 and a mass number of 27).

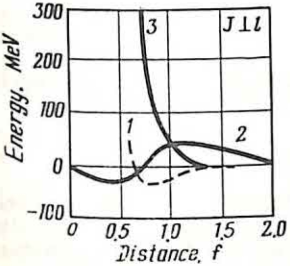


Fig. 87. Components of nuclear forces for identical nucleons (for the case when $l \perp J$): curve 1—central force; curve 2—spin-dependent force; curve 3—spin-orbit force

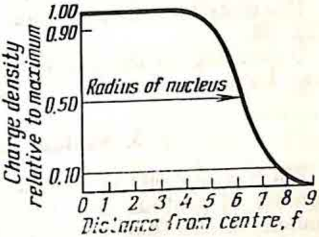


Fig. 88. Distribution of charge in a gold nucleus

The nuclear forces greatly change with a change in the distance; at a distance of 1 fermi the nuclear forces between protons are 35 times greater than the forces of electrical repulsion and 10^{38} times greater than gravitational interaction.

At distances less than 0.7 fermi the nuclear forces act like forces of repulsion, and at distances greater than 0.7 fermi, like forces of attraction; at a distance of 2 fermi their action vanishes. Nuclear forces do not depend on the electric charge and have the property of saturation.

Nuclear forces have three components: the central force depending on the distance, the spin-dependent force, and the spin-orbit force, which appears as a result of curving of the path of one nucleon when it passes near another nucleon. Curves of the potential energies corresponding to the components of the nuclear forces for identical nucleons are shown in Fig. 87 (for the case when $l \perp J$). The force equals the change in potential energy per unit of length.

The size and shape of a nucleus are assessed according to the dispersion of high-energy electrons.

The electric charge is distributed unevenly in a nucleus; in the core of a nucleus the density is constant; the core is surrounded by a shell in which the density diminishes. The *radius of a nucleus* is assumed to equal the distance from its centre to the layer in which the density of the charge equals one-half its maximum value.

The distribution of the charge in a gold nucleus is shown in Fig. 88.

Depending on the number of protons and neutrons, nuclei may have the shape of a sphere, cigar, or pear.

5. Nuclear Transformations

When a nucleus is formed by bringing together a certain number of protons and neutrons, the mass of the resulting nucleus is less than the sum of the masses of the component protons and neutrons. This difference is called the *mass defect* of the nucleus.

The energy released when neutrons and protons combine to form a nucleus is called the *binding energy* of the nucleus (E). In computations one usually deals with the quantity $\frac{E}{A}$, i.e. the binding energy per nucleon. The average value of the quantity $\frac{E}{A}$ for heavy nuclei is taken equal to 8×10^6 eV.

Some heavy nuclei (uranium, thorium, radium) spontaneously disintegrate with the formation of new nuclei and the emission of α-particles, electrons, and high-energy photons (γ-rays—see the electromagnetic spectrum). This property is called *natural radioactivity*.

Radioactivity obeys the following laws:

1. Radioactive decay does not depend on the external conditions (temperature, pressure, and chemical reactions).
2. α- and γ-particles have discrete values of energy; β-particles have different values of energy; β-decay is attended by the emission of a neutrino and an antineutrino.
3. The change in the number of radioactive nuclei is determined by the formula

$$N = N_0 e^{-\lambda t} = N_0 2^{-t/T} \tag{6.5}$$

where N_0 is the original number of nuclei present at the time $t = 0$, N—the number of nuclei remaining at the time t, T—the half-life—the time during which one-half of the nuclei disintegrate, and λ—the probability of the disintegration of one nucleus in one second (the *disintegration constant*).

The quantity $\tau = \frac{1}{\lambda}$ is called the *mean lifetime of a nucleus.*

4. The new nuclei formed after radioactive decay occupy other positions in the Periodic Table of elements: upon α-decay the atomic number diminishes by two, upon electron decay it grows by one, and upon positron decay it diminishes by one. This law is called the *displacement law.*

Artificial nuclear transformations can be induced by bombarding elements with protons, neutrons, nuclei of deuterium, helium or heavier elements, and also with γ-rays. This phenomenon is called *artificial radioactivity*. The transformations of nuclei in artificial radioactivity are called *nuclear reactions*. The latter may result in the formation of new radioactive isotopes that do not occur naturally on the Earth. In artificial radioactivity nuclei mainly emit β-particles and γ-rays.

The number of nuclear reactions K occurring in a substance during one second when it is bombarded with a particle flux P is determined by the formula

$$K = \sigma n_0 P \tag{6.6}$$

where n_0 is the number of atoms in the substance, and σ—a proportionality factor called the *effective cross section of a nuclear reaction*; this factor has the dimension of area and numerically equals the probability of a nuclear reaction occurring in one second when one nucleus is bombarded with a flux having a density of one particle per square centimetre per second.

The effective cross sections of nuclear reactions are measured in barns;

$$1 \text{ barn} = 10^{-24} \text{ cm}^2$$

The effective cross section for rapid particles is approximately equal to the cross-sectional area of a nucleus, i.e. $\sigma = \pi r_0^2$ (r_0 is the radius of the nucleus).

The average distribution of energy in the fission of one heavy nucleus is as follows:

kinetic energy of neutrons . . .	5 MeV
kinetic energy of fission products	165 MeV
energy of γ-radiation	8 MeV
energy carried off by a neutrino	11 MeV
energy of fission products (β-decay and γ-radiation) . . .	11 MeV

Nuclear transformations are used for the release of nuclear energy by the fission of heavy nuclei (for example, uranium U^{235}) or the synthesis (combination) of light nuclei (for example, hydrogen nuclei). Light nuclei can be synthesized at very high temperatures (of the order of magnitude of tens and hundreds of millions of kelvins); such reactions are called *thermonuclear*.

6. Wave Properties of Particles

Every moving particle has wave properties. For example, when an electron passes through a metal film, we obtain a diffraction pattern similar to the diffraction patterns of X-rays and γ-rays (Fig. 89).

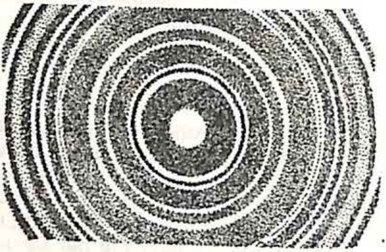


Fig. 89. Diffraction of electrons on thin silver foil

The wavelength of a particle is determined by the relation

$$\lambda = \frac{h}{mv} \tag{6.7}$$

where m is the mass of the particle, v—its velocity, and h—Planck's constant.

7. Interaction of Nuclear Radiation with a Substance

Electrons and positrons. The main kinds of interaction are elastic and inelastic scattering, and radiation deceleration.

Inelastic scattering is attended by ionization and excitation of atoms; these losses are called *ionization losses.*

When rapid electrons interact with nuclei, they are decelerated with the emission of braking radiation (bremsstrahlung), for example, X-rays. Such losses are called *radiation losses.*

The ionization and radiation losses determine the length of the path of an electron in a substance.

The absorption of electrons is described by the law

$$N_d = N_0 e^{-\mu d} \tag{6.8}$$

where N_0 is the initial intensity of the beam of electrons, N_d—the intensity of the beam after it passes through a layer of the substance with the thickness d, μ—the linear absorption coefficient (in cm^{-1}), $\frac{\mu}{\rho}$ —mass absorption coefficient which has approximately identical values in different substances (for one emitter), and ρ—the density of the substance.

Atomic nuclei. The radiation losses of atomic nuclei are not great; the main part is played by the ionization losses, which are determined by the charge and the velocity of the particles. For example, an electron and a proton have the same ionization losses at identical velocities. The length of the path of a particle, however (at the same energies) depends on its mass; it is smaller for heavy particles than for light ones. For α-particles the length of the path (in cm) in air (in standard conditions) is determined by the expression

$$R = 0.309 E^{3/2} \tag{6.9}$$

where E is the energy in MeV, changing from 4 to 7 MeV.

Neutrons. Neutrons interact with atomic nuclei by means of nuclear forces; two kinds of interaction are mainly distinguished—scattering and capture.

Scattering may be elastic and inelastic; in inelastic scattering the kinetic energy changes.

After several collisions neutrons are captured by the nuclei,

Gamma-radiation. The attenuation of the intensity of a beam of γ-rays is determined by the relationship

$$I = I_0 e^{-\mu d} \tag{6.10}$$

where I_0 is the intensity of the beam when it enters the layer of the substance, d—the thickness of the layer, μ—the linear attenuation factor, $\frac{\mu}{\rho}$—the mass attenuation factor.

The main kinds of interaction are scattering on electrons, photoelectric absorption, and the formation of pairs.

Two kinds of scattering are distinguished—coherent (for photons having an energy up to 10 keV) and incoherent (Compton). At high energies incoherent scattering predominates; it is attended by a change in the wavelength.

Photoelectric absorption is connected with the interaction of a photon with an electron of an atom. It has the greatest probability when the photon possesses an energy only slightly exceeding the binding energy of an electron.

A photon having an energy greater than $2mc^2$ (m is the mass of an electron, and c—the velocity of light) may transform near a nucleus into a pair of particles—an electron and a positron, which ionize the atoms of a substance.

Thus, the summary attenuation factor is

$$\mu = \sigma + \tau + \varkappa \tag{6.11}$$

where σ, τ, and $\varkappa$ are attenuation factors taking into account Compton scattering, photoelectric absorption, and the formation of electron-positron pairs, respectively.

8. Units of Radioactivity and Ionizing Radiation

The unit of activity of radioactive preparations—the curie (Ci)*—equals the activity of an isotope in which 3.7×10^{10} disintegrations occur a second. Submultiples of the curie are also used: the millicurie (10^{-3} Ci) and the microcurie (10^{-6} Ci). The mass of a radioactive element whose activity equals 1 curie can be calculated by the formula

$$M = 8.9 \times 10^{-14} AT \text{ g}$$

where A is the mass number, and T—the half-life (in s).

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* At present the activity is measured by the number of disintegrations a second (d/s).

The magnitude of radiation measured according to the ionization of air it produces is called the *irradiation dose* or the *radiation exposure* and is measured in roentgens. The roentgen (r) is a radiation exposure under whose action ions carrying one CGSE unit of charge are formed in 1 cm³ of dry air at a pressure of 760 mm Hg and a temperature of 0 °C. At a radiation exposure of 1 r the number of pairs of ions formed in a cm³ of air is 2.08×10^9.

The dose related to a unit of time is called the *power of the dose* or the *dose rate*. Its units are the roentgen/hour (r/h), roentgen/minute (r/min), and roentgen/second (r/s).

The energy of any kind of radiation absorbed by a unit of mass of the irradiated medium is called the *absorbed dose*. It is measured in rads. A rad is the absorbed radiation dose equal to 100 ergs per gramme of irradiated substance (erg/g).

A special unit—the man roentgen equivalent (rem) is used to take account of the different biological action of any kinds of radiation. 1 rem is the amount of energy absorbed by 1 g of live tissue when irradiated by any kind of ionizing radiation and causing the same biological effect as the absorption of a dose of 1 rad of X-ray or γ-radiation;

$$\text{dose (in rem)} = \text{dose (in rad)} \times \text{RBE}$$

where RBE is the relative biological effectiveness depending on the kind of radiation, the biological process being considered, and the power of the dose. Approximate values of RBE are given below:

X-rays and γ-rays, electrons	1
α-particles, protons	10-20
thermal neutrons	3
rapid neutrons	10
multiply charged ions	20

The RBE shows by how many times the biological action of a given kind of radiation differs from the action of X- or γ-rays.

9. Classification of Elementary Particles

Particles are divided into two kinds according to the magnitude of their spin—with an integral spin (0 and 1) and with a half-integral spin $\left(\frac{1}{2}\right)$. Particles with a half-integral spin obey the Pauli exclusion principle; particles with an integral spin do not obey this principle.

According to their mass particles are divided into three groups: *baryons* (heavy particles), *mesons*, and *leptons* (light particles).

A special antiparticle corresponds to each elementary particle. Any pair of such particles is characterized by three main properties:

1. A particle and the antiparticle corresponding to it have identical masses, spins and lifetimes.
2. A particle and its antiparticle have electric charges, baryon (lepton) numbers, and strangenesses that are equal in magnitude but opposite in sign.
3. The products of decay of a particle and the corresponding antiparticle are also particles and antiparticles with respect to one another.

The *baryon number* is a quantum number equal to $+1$ for nucleons and hyperons, -1 for their antiparticles, and to 0 for mesons and leptons.

The *lepton number* is introduced in a similar way; it equals $+1$ for leptons and -1 for their antiparticles.

Among the various elementary particles we can separate definite groups having close characteristics (spin, mass), but differing in their charge. Such groups are called *charge multiplets*; for example, a proton and a neutron, pi-mesons. The particles in such groups are considered as different states of a single particle.

The mean value of the charges of the particles in a multiplet is called the *centre of the charge multiplet*; for nucleons it equals $+\frac{1}{2}$, for pi-mesons it is 0.

The centres of the charge multiplets of hyperons are displaced relative to the corresponding centres of nucleons. *Strangeness* is a quantum number defined as the double value of the displacement of the centre of a charge multiplet. For pi-mesons and nucleons the strangeness equals zero.

10. Transformations of Particles

Owing to various kinds of interactions certain elementary particles transform into others. For example, upon weak interaction (which, however, is 10^{26} times greater than gravitational interaction) a neutron transforms into a proton, electron and antineutrino:

$$n \rightarrow p + e^- + \widetilde{\nu}$$

mesons disintegrate according to the schemes

$$\mu^+ \rightarrow e^+ + \nu + \widetilde{\nu}$$

$$\mu^- \rightarrow e^- + \widetilde{\nu} + \widetilde{\nu}$$

$$\pi^+ \rightarrow \mu^+ + \nu$$

$$\pi^- \rightarrow \mu^- + \widetilde{\nu}$$

Collisions of particles and antiparticles are attended by transformations according to the following schemes:

electron + positron → two gamma-quanta

i.e.

$$e^- + e^+ \rightarrow 2\gamma$$

proton + antiproton → five pi-mesons

i.e.

$$p + \widetilde{p} \rightarrow 2\pi^+ + 2\pi^- + \pi^0$$

In the deceleration of high-energy γ-quanta electron-positron pairs are formed:

$$\gamma \rightarrow e^- + e^+$$

The process of interaction of particles and antiparticles and their transformation into other particles is called *annihilation*. In annihilation of an electron and a positron particles having a rest mass form particles (γ-quanta) having no rest mass. Although the word "annihilation" means "destruction", no destruction of mass and energy occurs in these transformations, and the law of conservation of mass and energy is observed absolutely strictly. Typical examples of transformations of elementary particles are given in Table 139.

TABLES AND GRAPHS

Energy Levels of the Hydrogen Atom

The energy levels are calculated by formula (6.2), in which n is assumed to equal 1, 2, 3, 4, etc. Using an energy level diagram, it is easy to calculate the frequencies of the spectral lines of the hydrogen atom from Eq. (6.4).

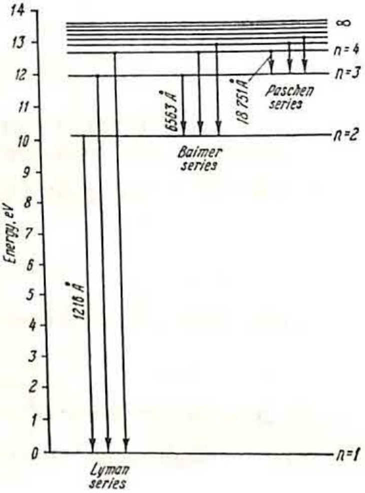

Fig. 90. Energy levels and emission spectrum of a hydrogen atom

Upon transition of the electrons to the level with $n = 1$ a system of lines called the *Lyman series* (Fig. 90) is emitted. The lines of this series are in the ultraviolet region of the spectrum. Upon transition to the level $n = 2$ the lines of the *Balmer series* are emitted (four lines of this series are in the visible part of the spectrum, and the remaining ones in the ultraviolet part). Upon transition to the level with $n = 3$ the lines of the *Paschen series* (in the infrared region) are emitted.

The numbers near the shortest arrows of each series in Fig. 90 indicate the maximum wavelength in Å in the given series (1 Å = 10^{-8} cm).

D. I. MENDELEEV'S PERIODIC

PERIODS	SERIES	ELEMENT				
		I	II	III	IV	V
1	I	**H** 1 1.0079 1				
2	II	**Li** 3 6.941 1 2	**Be** 4 9.0122 2 2	3 2 5 **B** 10.81	4 2 6 **C** 12.011	5 2 7 **N** 14.0067
3	III	**Na** 11 22.9898 1 8 2	**Mg** 12 24.305 2 8 2	3 8 2 13 **Al** 26.9815	4 8 2 14 **Si** 28.086	5 8 2 15 **P** 30.9738
4	IV	**K** 19 39.098 1 8 8 2	**Ca** 20 40.08 2 8 8 2	**Sc** 21 44.956 2 9 8 2	**Ti** 22 47.90 2 10 8 2	**V** 23 50.941 2 11 8 2
	V	1 18 8 2 29 **Cu** 63.546	2 18 8 2 30 **Zn** 65.38	3 18 8 2 31 **Ga** 69.72	4 18 8 2 32 **Ge** 72.59	5 18 8 2 33 **As** 74.9216
5	VI	**Rb** 37 85.468 1 8 18 8 2	**Sr** 38 87.62 2 8 18 8 2	**Y** 39 88.906 2 9 18 8 2	**Zr** 40 91.22 2 10 18 8 2	**Nb** 41 92.906 1 12 18 8 2
	VII	1 18 18 8 2 47 **Ag** 107.868	2 18 18 8 2 48 **Cd** 112.40	3 18 18 8 2 49 **In** 114.82	4 18 18 8 2 50 **Sn** 118.69	5 18 18 8 2 51 **Sb** 121.75
6	VIII	**Cs** 55 132.9054 1 8 18 18 8 2	**Ba** 56 137.34 2 8 18 18 8 2	**La*** 57 138.905 2 9 18 18 8 2	**Hf** 72 178.49 2 10 32 18 8 2	**Ta** 73 180.948
	IX	1 18 32 18 8 2 79 **Au** 196.967	2 18 32 18 8 2 80 **Hg** 200.59	3 18 32 18 8 2 81 **Tl** 204.37	4 18 32 18 8 2 82 **Pb** 207.2	5 18 32 18 8 2 83 **Bi** 208.9804
7	X	**Fr** 87 [223] 1 8 18 32 18 8 2	**Ra** 88 226.025 2 8 18 32 18 8 2	**Ac**** 89 [227] 2 9 18 32 18 8 2	**Ku** 104 [260] 2 10 32 32 18 8 2	

*LANTHA

Ce 58 140.12 2 8 20 18 8 2	**Pr** 59 140.908 2 8 21 18 8 2	**Nd** 60 144.24 2 8 22 18 8 2	**Pm** 61 [147] 2 8 23 18 8 2	**Sm** 62 150.4 2 8 24 18 8 2	**Eu** 63 151.96 2 8 25 18 8 2	**Gd** 64 157.25

**ACTI

Th 90 232.0381 2 10 18 32 18 8 2	**Pa** 91 231.036 2 9 20 32 18 8 2	**U** 92 238.029 2 9 21 32 18 8 2	**Np** 93 237.048 2 8 23 32 18 8 2	**Pu** 94 [242] 2 8 24 32 18 8 2	**Am** 95 [243] 2 8 25 32 18 8 2	**Cm** 96 [247]

.*Note*. The places in the table indicate the symbol of the el the element (the lower number). The columns of figures indicate th *O*, *P*, *Q*. The figures in brackets are the mass numbers of the mo

Table 132

TABLE OF ELEMENTS

GROUPS					
VI	VII	VIII			0
	(H)				**He** 2 4.0026 2
O 8 15.9994 6 2	**F** 9 18.9984 7 2				**Ne** 10 20.179 8 2
S 16 32.06 6 8 2	**Cl** 17 35.453 7 8 2				**Ar** 18 39.948 8 8 2
Cr 24 51.996 1 13 8 2	**Mn** 25 54.9380 2 13 8 2	**Fe** 26 55.847 2 14 8 2	**Co** 27 58.9332 2 15 8 2	**Ni** 28 58.71 2 16 8 2	
Se 34 78.96 6 18 8 2	**Br** 35 79.904 7 18 8 2				**Kr** 36 83.80 8 18 8 2
Mo 42 95.94 1 13 18 8 2	**Tc** 43 98.906 1 14 18 8 2	**Ru** 44 101.07 1 15 18 8 2	**Rh** 45 102.905 1 16 18 8 2	**Pd** 46 106.4 0 18 18 8 2	
Te 52 127.60 6 18 18 8 2	**I** 53 126.9045 7 18 18 8 2				**Xe** 54 131.30 8 18 18 8 2
W 74 183.85 2 12 32 18 8 2	**Re** 75 186.2 2 13 32 18 8 2	**Os** 76 190.2 2 14 32 18 8 2	**Ir** 77 192.22 2 15 32 18 8 2	**Pt** 78 195.09 1 17 32 18 8 2	
Po 84 [210] 6 18 32 18 8 2	**At** 85 [210] 7 18 32 18 8 2				**Rn** 86 [222] 8 18 32 18 8 2

NIDES

Tb 65 158.925 2 8 27 18 8 2	**Dy** 66 162.50 2 8 28 18 8 2	**Ho** 67 164.930 2 8 29 18 8 2	**Er** 68 167.26 2 8 30 18 8 2	**Tm** 69 168.934 2 8 31 18 8 2	**Yb** 70 173.04 2 8 32 18 8 2	**Lu** 71 174.97 2 9 32 18 8 2

NIDES

Bk 97 [247] 2 8 27 32 18 8 2	**Cf** 98 [249] 2 8 28 32 18 8 2	**Es** 99 [254] 2 8 29 32 18 8 2	**Fm** 100 [253] 2 8 30 32 18 8 2	**Md** 101 [256] 2 8 31 32 18 8 2	**No** 102 [254] 2 8 32 32 18 8 2	**Lr** 103 [257] 2 9 32 32 18 8 2

ment, the atomic number (the upper number), the atomic weight of number of electrons in the shells in the upward order *K*, *L*, *M*, *N*, stable or best studied (e.g. Po, Pm, and Cf) isotopes.

Table 138

Distribution of Electrons in Atom by Layers and Shells

Layer	Shell	Quantum numbers				Maximum number of electrons
		n	l	m	s	
K	1s	1	0	0	± 1/2	2
L	2s	2	0	0	± 1/2	2
	2p		1	−1 0 +1	± 1/2 ± 1/2 ± 1/2	6
M	3s	3	0	0	± 1/2	2
	3p		1	−1 0 +1	± 1/2 ± 1/2 ± 1/2	6
	3d		2	−2 −1 0 +1 +2	± 1/2 ± 1/2 ± 1/2 ± 1/2 ± 1/2	10
N	4s	4	0	0	± 1/2	2
	4p		1	−1 0 +1	± 1/2 ± 1/2 ± 1/2	6
	4d		2	−2 −1 0 +1 +2	± 1/2 ± 1/2 ± 1/2 ± 1/2 ± 1/2	10
	4f		3	−3 −2 −1 0 +1 +2 +3	± 1/2 ± 1/2 ± 1/2 ± 1/2 ± 1/2 ± 1/2 ± 1/2	14

Table 134

Main Lines of Characteristic X-Ray Spectrum of Selected Elements (*K*-Series)

Element	Wavelength, Å		
	α_2	α_1	β
Aluminium	9.89		9.56
Silicon	7.128	7.125	6.768
Chromium	2.294	2.290	2.085
Manganese	2.106	2.102	1.910
Iron	1.940	1.936	1.757
Cobalt	1.793	1.789	1.621
Nickel	1.662	1.658	1.500
Copper	1.544	1.541	1.392
Zinc	1.439	1.435	1.295
Germanium	1.258	1.254	1.129
Selenium	1.109	1.105	0.992
Tungsten	0.214	0.209	0.184
Gold	0.185	0.180	0.159
Lead	0.170	0.165	0.146
Uranium	0.131	0.126	0.111

Table 135

Selected Radioactive Isotopes and Their Characteristics

Element	Symbol of radioactive isotope	Half-life	Radiation energy, MeV	
			β-particles	γ-rays
Carbon	C_{6}^{14}	5568 yr	0.155	—
Sodium	Na_{11}^{24}	15.0 h	1.39	1.38, 2.76
Phosphorus	P_{15}^{32}	14.3 days	1.71	—
Sulphur	S_{16}^{35}	87.1 days	0.167	—
Chlorine	Cl_{17}^{36}	3.1×10^{5} yr	0.714	—
Calcium	Ca_{20}^{45}	152 days	0.254	—
Scandium	Sc_{21}^{46}	85 days	0.36	0.89, 1.12
Titanium	Ti_{22}^{51}	5.8 min	2.13	0.32, 0.61, 0.93
Chromium	Cr_{24}^{51}	28 days	—	0.32, 0.57
Iron	Fe_{26}^{55}	2.9 yr	—	0.21
	Fe_{26}^{59}	45 days	0.46, 0.27	1.1, 1.29
Cobalt	Co_{27}^{60}	5.3 yr	0.3	1.17, 1.33

Table 135 (concluded)

Element	Symbol of radio-active isotope	Half-life	Radiation energy, MeV	
			β-particles	γ-rays
Selenium .	Se^{75}_{34}	127 days	—	0.2-0.4
Strontium	Sr^{89}_{38}	51 days	1.46	—
Zirconium	Zr^{95}_{40}	65 days	0.36, 0.40, 0.88	0.23, 0.72, 0.75
Technetium	Tc^{99}_{43}	2.12×10⁵yr	0.3	—
Silver . . .	Ag^{110}_{47}	270 days	0.087, 0.53, 2.12, 2.86	0.1-2.5
	Ag^{111}_{47}	7.5 days	0.7, 0.8, 1.0	0.25, 0.34
Cadmium .	Cd^{109}_{48}	470 days	—	0.087
	Cd^{115}_{48}	43 days	0.7, 1.61	0.5-1.3
Antimony	Sb^{115}_{51}	2 yr	0.25, 0.34, 0.61, 0.82	0.08-0.72
Iodine . . .	I^{131}_{53}	8.14 days	0.25, 0.34, 0.61, 0.82	0.08-0.72
Cesium . .	Cs^{137}_{55}	27 yr	0.51, 1.17	0.66
Promethium	Pm^{147}_{61}	2.6 yr	0.22	0.121
Europium .	Eu^{155}_{63}	1.7 yr	0.15, 0.24	0.06-0.132
Thulium .	Tu^{170}_{69}	129 days	0.88, 0.97	0.08
Hafnium .	Hf^{181}_{72}	46 days	0.4	0.004-0.62
Tantalum	Ta^{182}_{73}	111 days	0.53	0.06-1.6
Tungsten .	W^{185}_{74}	73.2 days	0.37, 0.43	0.056, 0.57, 0.77
Iridium . .	Ir^{192}_{77}	74.4 days	0.10, 0.26, 0.54, 0.67	0.2-0.9
Gold . . .	Au^{198}_{79}	2.7 days	0.29, 0.96, 1.37	0.41-1.09
	Au^{199}_{79}	3.15 days	0.29, 0.44, 0.47	0.05, 0.16, 0.21
Mercury . .	Hg^{205}_{80}	5.6 min	1.8	0.23
Thallium .	Tl^{204}_{81}	4.1 yr	0.77	—

Table 136

Selected Artificially Produced Elements

Atomic number	Name	Symbol	Mass numbers of isotopes	Half-life
61	Promethium	Pm	146-151, **147**	2.6 yr
85	Astatine	At	210-219	8.3 h
87	Francium	Fr	219-223	23 min
93	Neptunium	Np	231-241, **237**	2.2×10^6 yr
94	Plutonium	Pu	236-246, **244**	7.5×10^7 yr
95	Americium	Am	240-246, **243**	8×10^3 yr
96	Curium	Cm	238-250, **247**	4×10^7 yr
97	Berkelium	Bk	243-250, **247**	700 yr
98	Californium	Cf	244-254, **251**	660 yr
99	Einsteinium	Es	247-255, **254**	appr. 1 yr
100	Fermium	Fm	250-256, **253**	4.5 yr
101	Mendelevium	Md	255, **256**	30 min
102	Nobelium	No	255	—
103	Lawrencium	Lr	257	appr. 8 s
104	Kurchatovium	Ku	260	appr. 0.3 s

Note. The mass number of the isotope with the greatest half-life is set in boldface type.

Table 137

Angular Momenta J and Magnetic Moments p of Nuclei

Z	Nucleus	Mass number	$J\ \left(\frac{h}{2\pi}\right)$	p (in nuclear magnetons)
0	n	1	1/2	−1.9125
1	H	1	1/2	2.7928
	D	2	1	0.8565
2	He	3	1/2	−2.131
		4	0	0
3	Li	6	1	0.821
		7	3/2	3.2532
4	Be	9	3/2	−1.176
5	B	11	3/2	2.686
6	C	12	0	0
		13	1/2	0.701
7	N	14	1	0.403
		15	1/2	0.280
8	O	16	0	0
17	Cl	35	5/2	1.368
80	Hg	199	1/2	0.5
		201	3/2	−0.6

Note. The minus sign indicates that the directions of the magnetic and mechanical moments are opposite.

Table 138

Elementary Particles

Particle	Symbol	Charge in e	Rest mass, MeV	Spin	Baryon number	Strangeness	Mean life-time, s	Usual products of decay
Hyperons:								
xi-minus	Ξ^-	−1	1319	1/2	+1	−2	2×10^{-10}	$\pi^-+\Lambda$
xi-zero	Ξ^0	0	1311	1/2	+1	−2	2×10^{-10}	$\pi^0+\Lambda$
sigma-minus	Σ^-	−1	1196	1/2	+1	−1	1.6×10^{-10}	π^-+n
sigma-zero	Σ^0	0	1192	1/2	+1	−1	10^{-20}	$\nu+\Lambda$
sigma-plus	Σ^+	+1	1190	1/2	+1	−1	0.8×10^{-10}	π^++n π^0+p
lambda	Λ	0	1115	1/2	+1	−1	2.5×10^{-10}	π^-+p
Nucleons:								
neutron	n	0	940	1/2	+1	0	1.0×10^{3}	$e^-+\nu+p$
proton	p	+1	938	1/2	+1	0	Stable	—
Mesons:								
K-zero	K^0	0	498	0	0	+1	1×10^{-10} or 6×10^{-8}	$\pi^++\pi^-$ $\pi^0+\pi^0$ $\pi^++\mu^-+\tilde{\nu}$
K-plus	K^+	+1	494	0	0	+1	1.2×10^{-8}	$\mu^++\nu$ $\pi^++\pi^0$
pi-plus	π^+	+1	140	0	0	0	2.6×10^{-8}	$\mu^++\nu$
pi-zero	π^0	0	135	0	0	0	10^{-15}	$\gamma+\gamma$
Photon	γ	0	0	1	0	0	Stable	—
Leptons:								
mu-minus-meson	μ^-	−1	106	1/2	0	—	2.26×10^{-6}	$e^-+\nu+\tilde{\nu}$
electron	e^-	−1	0.511	1/2	0	—	Stable	—
neutrino	ν	0	0	1/2	0	—	Stable	—

Table 139

Sources of Radiation and Detectors for Tracing Elementary Particles

Particle	Source of radiation	What was measured or observed	Method of observation
$\widetilde{\nu}$	Nuclear reactor	$p+\widetilde{\nu}=n+e^+$, annihilation of e^+, capture of n	Counters
e^-	Cathode-ray tube	Ratio of charge to mass	Fluorescent screen
e^+	Cosmic rays	Ditto	Wilson cloud chamber
μ^+, μ^-	Ditto	No absorption of radiation when passing through lead was observed; disintegrates in state of rest	Ditto
π^+	Ditto	Disintegration in state of rest	Emulsion
π^-	Ditto	Nuclear interaction in state of rest	Ditto
π^0	Accelerator	Disintegration with formation of neutrino	Counters
K^+	Cosmic rays	Disintegration	Emulsion
K^-	Ditto	Nuclear interaction in state of rest	Ditto
K^0	Ditto	Disintegration into $\pi^++\pi^-$	Wilson cloud chamber
n	Polonium-beryllium source	Mass	Ionization chamber
$\widetilde{p}$	Accelerator	Ratio of charge to mass and annihilation	Counters
$\widetilde{n}$	Ditto	Annihilation	Ditto
Λ^0	Cosmic rays	Disintegration into $p+\pi^-$	Wilson cloud chamber
$\widetilde{\Lambda}^0$	Accelerator	Disintegration into $p+\pi^+$	Emulsion
Σ^+	Cosmic rays	Disintegration in state of rest	Ditto
Σ^-	Accelerator	Disintegration into π^-+n	Diffusion chamber
Σ^0	Ditto	Disintegration into $\Lambda^0+\gamma$	Bubble chamber
Ξ^-	Cosmic rays	Disintegration into $\pi^-+\Lambda^0$	Wilson cloud chamber
Ξ^0	Accelerator	Disintegration into $\pi^0+\Lambda^0$	Bubble chamber
ν	Ditto	$\pi \longrightarrow \mu^++\nu$ $\nu+n \longrightarrow p+\mu^-$	Spark chamber

Table 140

Effective Cross Sections of Neutrons

Element	Fast neutrons	Thermal neutrons		
		σ_{sc}	σ_{ab}	σ_{act}
H	0.9	38 (H_2)	0.33	—
He	1.4	0.8	—	—
Al	1.7	1.4	0.23	0.23
Fe	3.0	11.4	2.53	0.003
Ni	3.2	17.5	4.6	0.03
Cu	3.2	7.8	3.7	0.64, 2.9
Ge	3.4	9	2.4	0.002, 0.02. 0.2, 0.6
Cd	4.3	7	2600	0.1, 0.3, 0.04
Hg	4.8	21	380	0.025, 1.0
Pb	4.7	11.4	0.17	0.0003
Th-232 . .	7.2	12.6	7.4	7.4
U	5.2	8.3	7.68	2.73, 0.76
U-235 .	1.3	—	687	107, 580 (fission)
Pu-239	2.0	—	1065	315, 750 (fission)

Note. The first column indicates the element; the second—the total cross section of absorption and scattering of fast neutrons (3-10 MeV); columns 3-5 indicate the effective cross sections (in barns) for thermal neutrons (0.025 eV): σ_{sc} is the scattering cross section, σ_{ab}—the absorption cross section, and σ_{act}—the cross section of formation of an artificial element (the activation section) for various isotopes. For uranium-235 and plutonium-239 the effective cross sections of the fission reaction are indicated.

Table 141

Mass Absorption Coefficients of X-Rays

$$\left(\frac{\mu}{\rho},\ \ \mathrm{cm^2/g}\right)$$

Wavelength, Å	Elements							
	C	N	O	Al	Fe	Cu	Ag	Pb
0.2	0.167	0.177	0.183	0.27	1.06	1.45	5.4	4.6
0.4	0.243	0.34	0.336	1.05	7.1	10	37	33
0.6	0.40	0.73	0.730	3.3	23.5	32	17	77
0.8	0.80	1.51	1.53	7.3	50.7	71	39	147
1.0	1.40	2.6	—	14.0	95	134	71	77
1.2	2.5	—	—	24	170	218	120	128
1.4	3.9	—	—	36	270	42	174	180
1.6	5.8	—	—	55	390	60	250	258
1.8	7.9	—	—	79	61	85	354	360
2.0	10.0	—	—	106	78	119	436	—

Table 142

Mass Absorption Coefficients of Electrons in Aluminium

Energy, eV	$\frac{\mu}{\rho}$, cm²/g	Energy, eV	$\frac{\mu}{\rho}$, cm²/g
9×10^2	2.5×10^6	1.0×10^5	1.3×10^2
5.8×10^3	1.5×10^5	2.0×10^5	29
1.05×10^4	3.5×10^4	4.6×10^5	9.0
4.66×10^4	7.4×10^2	6.6×10^5	6.0

Table 143

Maximum Permissible Doses of Radiation

The danger of internal radiation depends on what organ is affected. A critical organ is one in which the accumulation of a radioactive isotope affects the entire organism to a greater extent. Sanitary rules establish three groups of critical organs: group I—the entire body, the gonads, crystalline lens, and blood-forming organs; group II—the muscles, adipose tissue, liver, kidneys, pancreas, prostate gland, gastro-intestinal tract, and lungs; group III—the skin, thyroid gland, and bones.

Category of radiation	External radiation		Internal radiation					
			Group I		Group II		Group III	
	mrem/week	rem/year	mrem/week	rem/year	mrem/week	rem/year	mrem/week	rem/year
A—occupational	100	5	100	5	300	15	600	30
B—in adjacent premises . . .	10	0.5	10	0.5	30	1.5	60	3
C—irradiation of entire population . . .	1	0.05	1	0.05	10	0.5	20	1

Note. In all cases by an age of 30 years the total dose should never exceed 60 rem.

Table 144

Path of α-Particles in Air, Biological Tissue, and Aluminium

Energy, MeV	In air, cm	In tissue, μm	In aluminium, μm
4.0	2.5	31	16
5.0	3.5	43	23
6.0	4.6	56	30
7.0	5.9	72	38
8.0	7.4	91	48
9.0	8.9	110	58
10.0	10.6	130	69

Components of Complete Attenuation of γ-Rays in Lead and Aluminium

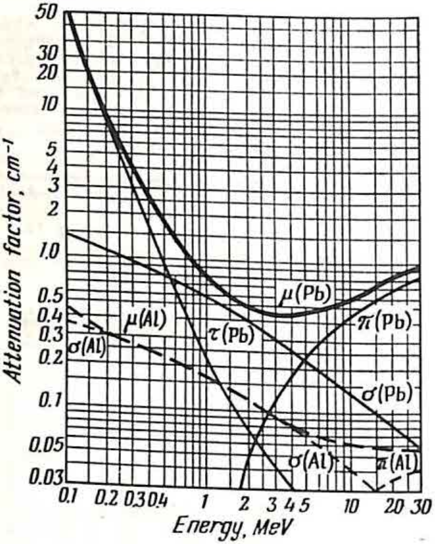

Fig. 91. Complete attenuation factor of gamma-rays $\mu = \sigma + \tau + \varkappa$, where σ, τ, and $\varkappa$ are attenuation factors taking into account Compton scattering, photoelectric absorption, and the formation of electron-positron pairs, respectively, in lead (Pb) and aluminium (Al). Logarithmic scales are used along both axes

Binding Energy of Nuclei

The mass numbers are laid off along the axis of abscissas (Fig. 92), and the values of the binding energy per nucleon in MeV along the axis of ordinates. The values of the binding energy for light nuclei are given in the form of separate points; the solid curve is the averaged values by isobars; the peaks and valleys in the curve are observed in experiments.

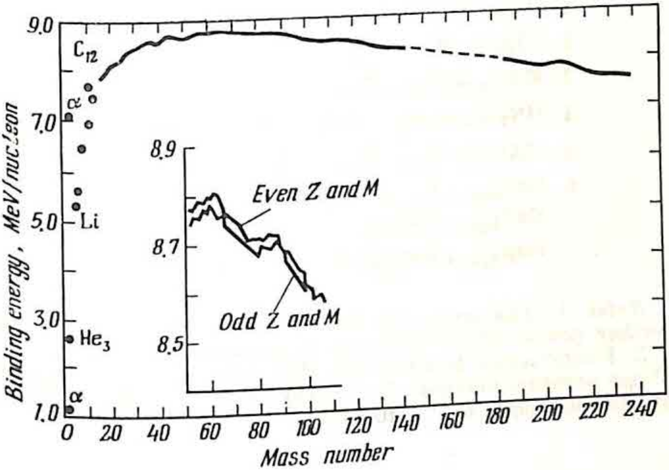

Fig. 92. Curve of binding energy of nuclei

The inside graph shows the experimental values for nuclei with mass numbers from 50 to 100 (the scale along the axis of ordinates has been increased).

Nuclear energy can be released either by the fission of heavy nuclei or by the synthesis of light ones. In both cases new nuclei are produced with a greater binding energy than that before the transformation.

Examples of Nuclear Reactions

Nuclear reactions proceed with the liberation or absorption of energy.

In the reactions given below, the numbers in the left-hand sides of the reactions indicate the magnitude of the absorbed energy, and in the right-hand sides—of the liberated energy in MeV:

1. ${}^{14}N_7 + {}^{4}He_2 + 1.1 = {}^{17}O_8 + {}^{1}H_1$.
2. ${}^{17}Li_3 + {}^{1}H_1 = 2\,{}^{4}He_2 + 17.3$.
3. ${}^{27}Al_{13} + {}^{4}He_2 + 28.2 = {}^{30}P_{15} + {}^{1}n_0$.
4. ${}^{14}N_7 + n = {}^{14}C_6 + {}^{1}H_1 + 0.6$.
5. ${}^{27}Al_{13} + {}^{12}C_6 = {}^{34}Cl_{17} + {}^{4}He_2 + {}^{1}n_0 + 28.2$.
6. ${}^{232}Th_{90} + {}^{1}n_0 \rightarrow {}^{233}Th_{90}$,
   ${}^{233}Th_{90} \rightarrow {}^{233}Pa_{91} + e^-$,
   ${}^{233}Pa_{91} \rightarrow {}^{233}U_{92} + e^-$.

Notes. 1. The arrows in the equations indicate that the further course of the reaction is spontaneous.

2. Energy equal to about 200 MeV is released in the fission of one uranium nucleus. The energy released by one gramme of uranium equals 2 × 10^3 kWh.

Synthesis of Helium from Hydrogen

The production of helium nuclei by the synthesis of hydrogen nuclei is of immense theoretical and practical importance, because a large amount of energy is released.

Some of the possible reactions are given below. The magnitude of the energy released is given in MeV.

1. ${}^{2}H_1 + {}^{2}H_1 \rightarrow {}^{4}He_2 + \gamma + 23.8$.
2. ${}^{2}H_1 + {}^{1}H_1 \rightarrow {}^{3}He_2 + \gamma + 5.4$.
3. ${}^{3}He_2 + {}^{1}H_1 \rightarrow {}^{4}He_2 + e^+ + 18.7$,
   ${}^{3}He_2 + {}^{1}H_3 \rightarrow {}^{4}He_2 + {}^{2}H_1 + 14.3$.

Reactions of Nuclear Synthesis

Reactions of nuclear synthesis can proceed only at high temperatures (scores of millions of kelvins). Some examples of such reactions are given below.

1. ${}^{12}C_6 + {}^{1}H_1 \rightarrow {}^{13}N_7 + \gamma + 1.9.$
2. ${}^{13}N_7 \rightarrow {}^{13}C_6 + e^+ + 1.2.$
3. ${}^{13}C_6 + {}^{1}H_1 \rightarrow {}^{14}N_7 + 7.5.$
4. ${}^{14}N_7 + {}^{1}H_1 \rightarrow {}^{15}O_8 + \gamma + 7.3.$
5. ${}^{15}O_8 \rightarrow {}^{15}N_7 + e^+ + 1.7.$
6. ${}^{15}N_7 + {}^{1}H_1 \rightarrow {}^{12}C_6 + {}^{4}He_2 + 4.9.$

APPENDICES

I. Some Frequently Encountered Numbers

$\pi = 3.141\,593$

$4\pi = 12.566\,37$

$\frac{2}{\pi} = 0.636\,62$

$\pi^2 = 9.869\,60$

$\sqrt{\pi} = 1.772\,45$

$e = 2.718\,282$

$\sqrt{2} = 1.414\,21$

$\sqrt{3} = 1.732\,05$

$1^\circ = 0.017\,453$ rad

$1' = 0.000\,291$ ”

$1'' = 0.000\,004\,8$ ”

II. Formulas for Approximate Calculations

$$\frac{1}{1+x} = 1 - x \qquad -0.031 < x < 0.031$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x \qquad -0.085 < x < 0.093$$

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x \qquad -0.052 < x < 0.052$$

$$\sin x = x \qquad -0.077 < x < 0.077$$

$$e^x = 1 + x \qquad -0.045 < x < 0.045$$

The inequalities indicate the range of values of x for which the error of the computations by the approximate formulas does not exceed 0.1%.

III. Elements of the Theory of Errors

All measurements can be performed only up to a certain degree of precision.

Precision is determined by the last significant figure of the measurement. To exclude chance errors which are always possible, the measurement should be repeated several times and the arithmetic mean of all the results taken.

If a quantity A has been measured n times and $A_1, A_2, \ldots, A_n$ are the results of the individual measurements, then the *arithmetic mean* is

$$A_{\text{mean}} = \frac{A_1 + A_2 + \ldots + A_n}{n}$$

The deviation $\Delta A_i = |A_{\text{mean}} - A_i|$ is called the *absolute error* of a single measurement. The quantity

$$\Delta A = \frac{\Delta A_1 + \Delta A_2 + \ldots + \Delta A_n}{n}$$

is called the *mean absolute error* of the measurement.

Usually it is considered that $A_{\text{mean}} - \Delta A < A < A_{\text{mean}} + \Delta A$.

The ratio $\frac{\Delta A}{A_{\text{mean}}}$ is called the *mean relative error* of the measurement and is usually expressed in per cent.

The result sought by the experimenter is rarely found by measuring only one quantity. It is generally necessary to measure several quantities and to calculate the desired result by a formula. The following table gives expressions for finding the absolute and relative errors of calculations carried out by some frequently encountered formulas.

Formula	Absolute error	Relative error	Formula	Absolute error	Relative error
$A + B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{\lvert A + B \rvert}$	$\frac{A}{B}$	$\frac{B\,\Delta A + A\,\Delta B}{B^2}$	$\frac{\Delta A}{\lvert A \rvert} + \frac{\Delta B}{\lvert B \rvert}$
$A - B$	$\Delta A + \Delta B$	$\frac{\Delta A + \Delta B}{\lvert A - B \rvert}$	A^n	$nA^{n-1}\,\Delta A$	$n\frac{\Delta A}{\lvert A \rvert}$
AB	$A\,\Delta B + B\,\Delta A$	$\frac{\Delta A}{\lvert A \rvert} + \frac{\Delta B}{\lvert B \rvert}$	$\sqrt[n]{A}$	$\frac{1}{n}A^{\frac{1-n}{n}}\,\Delta A$	$\frac{1}{n}\frac{\Delta A}{\lvert A \rvert}$
$\sin A$	$\lvert \cos A \rvert\,\Delta A$	$\lvert \cot A \rvert\,\Delta A$	$\cos A$	$\lvert \sin A \rvert\,\Delta A$	$\lvert \tan A \rvert\,\Delta A$

Example. To determine the density of a solid one measures its volume and its mass. Assume that the volume has been measured with an accuracy of 1.5%, and the mass with an accuracy of 1%. Then the relative error of the determined value of the density is 2.5%. Hence, we may write:

$$\left(\frac{m}{V}\right)_{\text{mean}}(1-0.025)<\frac{m}{V}<\left(\frac{m}{V}\right)_{\text{mean}}(1+0.025)$$

IV. Prefixes for Multiples and Submultiples of Units

Tera (T)	10^{12}	Centi (c)	10^{-2}
Giga (G)	10^{9}	Milli (m)	10^{-3}
Mega (M)	10^{6}	Micro (μ)	10^{-6}
Kilo (k)	10^{3}	Nano (n)	10^{-9}
Deci (d)	10^{-1}	Pico (p)	10^{-12}

V. Units of Selected Physical Quantities

Mass

1 ton = 10 centners = 1000 kg
1 carat = 2×10^{-4} kg
1 pood = 16.380 50 kg

Length

1 μm (micron) = 10^{-6} m
1 Å (angstrom) = 10^{-8} cm
1 X (x-unit) = 10^{-11} cm
1 inch = 25.40 mm
1 foot = 0.304 80 m
1 yard = 0.914 40 m

1 mile (U.K.) = 1609 m
1 nautical mile = 1852 m
1 light year = 9.463×10^{12} km
1 parsec = 3.26 light years = 3.08×10^{13} km

Time

1 year = 31 556 925.975 s
1 day = 24 h = 1440 min = 86 400 s
1 h = 60 min = 3600 s

Pressure

1 at (technical) = 1 kgf/cm^2 = 735.66 mm Hg = $9.806\ 65 \times 10^4$ N/m^2
1 mm Hg (millimetre of mercury) = 0.001 316 at = 1333 dyn/cm^2 = 133.3 N/m^2
1 atm (standard atmosphere) = 760 mm Hg = 1.033 kgf/cm^2 = 1.013×10^6 dyn/cm^2 = 1.013×10^5 N/m^2

Temperature

Number of degrees Celsius (°C) $= \frac{5}{4}$ °R $= \frac{5}{9}$(°F − 32) = (K − 273). Here °R stands for the number of degrees on the Réaumur scale, °F—the number of degrees Fahrenheit, and K—the number of kelvins.

Thermal Conductivity

1 kcal/m·h·K = 2.778×10^{-3} cal/cm·s·K = 1.162×10^{-2} W/cm·K

Work and Energy

1 W·h = 3600 J
1 J (joule) = 10^{7} erg = 0.239 cal (calorie)
1 kgf·m [kilogram(force)-metre] = 9.81 J
1 kcal (kilocalorie) = 1.16 W·h
1 eV (electron-volt) = 1.6019×10^{-12} erg = 1.6019×10^{-19} J

Power

1 W = 10^{7} erg/s, 1 kW = 102 kgf·m/s = 1.36 hp (metric horsepower)

Capacitance

1 cm = 1.11 pF = 1.11×10^{-12} F

VI. Universal Physical Constants

Gravitational constant γ	6.67×10^{-11} $\mathrm{kg^{-1} \cdot m^{3} \cdot s^{-2}}$
Volume of a mole of an ideal gas in standard conditions V_μ	22.4207 l
Molar gas constant R	8.314 $\mathrm{J \cdot K^{-1} \cdot mol^{-1}}$
Faraday constant F	96 520 $\mathrm{C \cdot g\text{-}eq^{-1}}$
Avogadro constant N_A	6.0247×10^{23} $\mathrm{mol^{-1}}$
Boltzmann constant k	1.3805×10^{-16} $\mathrm{erg \cdot K^{-1}}$
Mass of a hydrogen atom m_H	1.673×10^{-24} g
Mass of a proton m_p	1.672×10^{-24} g
Mass of an electron m_e	9.108×10^{-28} g
Charge of an electron e	4.8029×10^{-10} CGSE 1.602×10^{-19} C
Velocity of light in a vacuum c	$2.997\,93 \times 10^{10}$ $\mathrm{cm \cdot s^{-1}}$
Wien constant c_1	0.289 780 cm·K
Planck constant h	6.6254×10^{-27} erg·s
Stefan-Boltzmann constant σ	5.6696×10^{-12} $\mathrm{W \cdot cm^{-2} \cdot K^{-4}}$
Rydberg constant for hydrogen R_H	109 677.576 $\mathrm{cm^{-1}}$
Rydberg constant for deuterium R_D	109 707.419 $\mathrm{cm^{-1}}$
Bohr magneton μ_B	9.273×10^{-21} erg/Gs
Nuclear magneton μ_n	5.050×10^{-24} erg/Gs

VII. SI System of Units

The following table gives the names, symbols, and dimensions of the most frequently used units of the SI system.

For conversion to other systems—CGSE and CGSM—the last two columns give the relevant conversion factors.

The CGSE and CGSM systems completely coincide for mechanical quantities; their basic units are the centimetre, gramme (mass), and second.

The two systems differ for electrical quantities. This is due to the circumstance that the fourth basic unit in the CGSE system is the permittivity of a vacuum ($\varepsilon_0 = 1$), and in the CGSM system the permeability of a vacuum ($\mu_0 = 1$).

In the Gaussian system of units the basic units are the centimetre, gramme, second, $\varepsilon_0 = 1$, and $\mu_0 = 1$ (for a vacuum). In this system electrical quantities are measured in the CGSE, and magnetic quantities in the CGSM system.

Quantity	Name	Dimension	Symbol	Conversion factors from SI system to	
				CGSE	CGSM
		1. Basic Units			
Length	metre	m	m	10^2 cm	10^2 cm
Mass	kilogram	kg	kg	10^3 g	10^3 g
Time	second	s	s	1 s	1 s
Current	ampere	A	A	3×10^9	10^{-1}
Temperature	kelvin	K	K	—	—
Luminous intensity .	candela	cd	cd	—	—

(Continued)

Quantity	Name	Dimension	Symbol	Conversion factors from SI system to	
				CGSE	CGSM
		2. Mechanical Units			
Velocity	metre per second	m/s	m/s	10^2 cm/s	10^2 cm/s
Acceleration	metre per second per second	m/s^2	m/s^2	10^2 cm/s^2	10^2 cm/s^2
Energy and work . .	joule	$kg \cdot m^2/s^2 = J$	J	10^7 erg	10^7 erg
Force	newton	$kg \cdot m/s^2 = J/m$	N	10^5 dyn	10^5 dyn
Power	watt	$kg \cdot m^2/s^3 = J/s$	W	10^7 erg/s	10^7 erg/s
		3. Electrical Units			
Quantity of electricity	coulomb	$A \cdot s = C$	C	3×10^9	10^{-1}
Voltage, e.m.f. . . .	volt	$kg \cdot m^2/A \cdot s^3 = V$	V	$\frac{1}{300}$	10^8
Electric field intensity	volt per metre	$kg \cdot m/A \cdot s^3 = V/m$	V/m	$\frac{1}{3} \times 10^{-4}$	10^6

(Concluded)

Quantity	Name	Dimension	Symbol	Conversion factors from SI system to	
				CGSE	CGSM
Capacitance	farad	$A^2 \cdot s^4/kg \cdot m^2 = A \cdot s/V = s/\Omega$	F	9×10^{11} cm	10^{-9}
Resistance	ohm	$kg \cdot m^2/A^2 \cdot s^3 = V/A$	Ω	$\frac{1}{9} \times 10^{-11}$	10^9
Resistivity	ohm per metre	$kg \cdot m^3/A^2 \cdot s^3 = \Omega \cdot m$	$\Omega \cdot m$	$\frac{1}{9} \times 10^{-9}$	10^{11}
Permittivity	farad per metre	$A^2 \cdot s^4/kg \cdot m^3 = F/m$	F/m	$36\pi \times 10^9$	$4\pi \times 10^{-11}$
		4. Magnetic Units			
Magnetic flux	weber	$kg \cdot m^2/A \cdot s^2$	Wb	$\frac{1}{300}$	10^8 Mx
Magnetic induction .	tesla	$kg/A \cdot s^2$	T	$\frac{1}{3} \times 10^{-6}$	10^4 Gs
Magnetic field intensity	ampere per metre	A/m	A/m	$12\pi \times 10^7$	$4\pi \times 10^{-3}$ Oe
Inductance	henry	$kg \cdot m^2/A^2 \cdot s^2 = \Omega \cdot s$	H	$\frac{1}{9} \times 10^{-11}$	10^9 cm
Permeability	henry per metre	H/m	H/m	$\frac{1}{36\pi \times 10^{13}}$	$\frac{1}{4\pi} \times 10^7$
		5. Optical Units			
Luminous flux . . .	lumen	cd·sr	lm	—	—
Luminance	nit	cd/m	nt	—	—
Illuminance	lux	lm/m^2	lx	—	—

VIII. Fundamental Equations of Electromagnetism Written in the SI and CGS (Gaussian) Systems of Units

<table>
<tr><th>Law or quantity</th><th>SI</th><th>CGS</th></tr>
<tr><td>Coulomb's law . . .</td><td>$F=\frac{q_1q_2}{4\pi\varepsilon\varepsilon_0r^2}$</td><td>$F=\frac{q_1q_2}{\varepsilon r^2}$</td></tr>
<tr><td>Intensity of field of point charge .</td><td>$E=\frac{q}{4\pi\varepsilon\varepsilon_0r^2}$</td><td>$E=\frac{q}{\varepsilon r^2}$</td></tr>
<tr><td>Intensity of field of plane capacitor . . . .</td><td>$E=\frac{\sigma}{\varepsilon\varepsilon_0}$</td><td>$E=\frac{4\pi\sigma}{\varepsilon}$</td></tr>
<tr><td>Force acting on charge in electric field . . . .</td><td colspan="2">$F=qE$</td></tr>
<tr><td>Work of displacement of charge in electric field . . .</td><td colspan="2">$A=qU$</td></tr>
<tr><td>Capacitance . . . .</td><td colspan="2">$C=\frac{q}{U}$</td></tr>
<tr><td>Capacitance of plane capacitor .</td><td>$C=\frac{\varepsilon\varepsilon_0S}{d}$</td><td>$C=\frac{\varepsilon S}{4\pi d}$</td></tr>
<tr><td>Moment of electric dipole . . . . . .</td><td colspan="2">$p=ql$</td></tr>
<tr><td>Density of electric field energy . . .</td><td>$w_e=\frac{\varepsilon\varepsilon_0E^2}{2}$</td><td>$w_e=\frac{\varepsilon E^2}{8\pi}$</td></tr>
<tr><td>Ohm's law . . . . .</td><td colspan="2">$i=\frac{U}{R}$</td></tr>
<tr><td>Power of electric current . . . . . .</td><td colspan="2">$P=iU$</td></tr>
<tr><td>Electrical resistance . . . .</td><td colspan="2">$R=\rho\frac{l}{S}$</td></tr>
<tr><td>Interaction of long parallel conductors carrying a current . . . . . .</td><td>$F=\frac{\mu\mu_0i_1i_2l}{2\pi r}$</td><td>$F=\frac{1}{c^2}\frac{2\mu i_1i_2l}{r}$</td></tr>
<tr><td>Magnetic flux . . .</td><td colspan="2">$\Phi=BS$</td></tr>
<tr><td>Biot and Savart's law . . . .</td><td>$\Delta H=\frac{i\Delta l\sin\alpha}{4\pi r^2}$</td><td>$\Delta H=\frac{1}{c}\frac{i\Delta l\sin\alpha}{r^2}$</td></tr>
</table>

(Concluded)

Law or quantity	SI	CGS
Intensity of magnetic field of forward current . .	$H = \frac{i}{2\pi r}$	$H = \frac{1}{c}\frac{2i}{r}$
Ampere's law . . .	$\Delta F = Bi\,\Delta l \sin\alpha$	$\Delta F = \frac{1}{c} Bi\,\Delta l \sin\alpha$
Lorentz force . . .	$F = qvB\sin\alpha$	$F = \frac{1}{c} qvB\sin\alpha$
Law of electromagnetic induction	$\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$	$\mathcal{E} = -\frac{1}{c}\frac{\Delta\Phi}{\Delta t}$
Inductance of solenoid	$L = \frac{\mu\mu_0 N^2 S}{l}$	$L = \frac{4\pi\mu N^2 S}{l}$
Density of magnetic field energy	$w_{\mathrm{m}} = \frac{\mu\mu_0 H^2}{2}$	$w_{\mathrm{m}} = \frac{\mu H^2}{8\pi}$
Thomson formula	$T = 2\pi\sqrt{LC}$	$T = \frac{2\pi}{c}\sqrt{LC}$
Reactance	$R_{L,\,C} = \omega L - \frac{1}{\omega C}$	$R_{L,\,C} = \frac{\omega L}{\varepsilon^2} - \frac{1}{\omega C}$
Density of electromagnetic wave flux	$S = EH$	$S = \frac{c}{4\pi} EH$
Velocity of light	$v = \frac{1}{\sqrt{\mu\mu_0\varepsilon\varepsilon_0}}$	$v = \frac{c}{\sqrt{\varepsilon\mu}}$

Note. Equations having the same form in both systems are given once (in the middle of the two columns).

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